Strouhal Number and Optimization of Swimming by Odontocete Cetaceans

Jim Rohr
SSC San Diego

Frank Fish
West Chester University

BACKGROUND

An important aspect of swimming is the ability to move efficiently. Paradoxically, early attempts at building fish-inspired mechanisms achieved disappointingly low efficiencies. It was only through a deeper understanding of the vorticity produced along the swimming animal and in its wake that significant progress was achieved. Beginning almost 40 years ago at a precursor to this laboratory, Moe Rosen discerned [1, 2], through a series of innovative flow visualization experiments, a system of vortices appearing along the sides of swimming fish and dolphin. Rosen hypothesized that some of the rotational energy surrounding the undulating motion of a fish or dolphin could be regained for propulsion through proper synchronization of the animal's body to the vortex flow.

A similar conclusion, but obtained through more rigorous theoretical analysis and detailed experimental studies, has been drawn by Triantafyllou et al. [3, 4, 5]. Performing stability analysis of the average velocity profiles of a pitching airfoil, Triantafyllou et al. [3, 4] have shown that maximum spatial amplification and optimum creation of thrust-producing jet vortices lie in a narrow range of nondimensional frequencies referred to as the Strouhal number, St. The predicted St range for maximum spatial amplification occurs between 0.25 and 0.35, peaking at 0.30. It was subsequently argued that the swimming efficiency for fish and cetaceans would also peak at $St = 0.25$ to 0.35.

The Strouhal number, St, which is related to how fast vortices are being generated and the space between them, is defined as:

$$St = A f / U,$$

where A is the width of the wake, taken here to be equal to the peak-to-peak maximum excursion of the trailing edge of the foil or fluke; f is the frequency of oscillation, and U is the average forward velocity. Similar combinations of these kinematic swimming parameters have previously been made to characterize the swimming motion of fish, cetaceans, and athletes, but not within such a predictive theoretical framework.

Although many species of cetaceans are believed to be particularly swift and efficient swimmers, corresponding St data have been extremely limited. The cetacean Strouhal number data of Triantafyllou et al. [4] consist of only two values. Moreover, the data were indirectly derived from analysis of traces [6] obtained from motion-picture frames of a 2.03-m Pacific white-sided dolphin (*Lagenorhynchus obliquidens*) swimming non-uniformly within a tank. One St value, 0.32, corresponded to the dolphin

ABSTRACT

Swimming efficiencies of fish and cetaceans have been related to a certain synchrony between stroke-cycle frequency, peak-to-peak tail/fluke amplitude, and mean swimming speed. These kinematic parameters form a nondimensional wake parameter, referred to as a Strouhal number, which, for the range between 0.20 to 0.40, has been associated with enhanced swimming efficiency. Yet, there has been no direct experimental substantiation as to what Strouhal numbers swimming cetaceans prefer. To address this lack of data, Strouhal numbers were calculated for several species of captive odontocete cetaceans. Although the average Strouhal number calculated for each species is within the accepted range, a greater proportion of Strouhal values (n = 248) occurs between the 0.20 to 0.30 (74%) range than the 0.25 to 0.35 (55%) range predicted for maximum swimming efficiency. Fluke-beat frequency normalized by the ratio of swimming speed to body length was generally restricted from 1 to 2, whereas peak-to-peak fluke amplitude normalized by body length occurred predominantly between 0.15 and 0.25. These results indicate that the Strouhal number range for odontocete cetaceans occurs at slightly lower values (~20%) than previously predicted for maximum swimming efficiency, and that the kinematics of the propulsive flukes of odontocete cetaceans are not solely dependent on Strouhal number.

swimming while wearing a 1.91-cm-diameter drag collar. The remaining *St* of 0.30 corresponded to swimming without the drag collar. References to this less-than-optimal data set appear repeatedly throughout the literature.

Many different species of captive odontocete cetaceans have been trained to swim steadily behind viewing panels and provide a unique opportunity for a much larger, more accurate *St* database. Strouhal numbers presented here are calculated from video recordings of six species of trained odontocete cetaceans obtained at Sea World Orlando, FL; Sea World San Antonio, TX; and Sea World San Diego, CA. The species include members of the family Delphinidae: bottlenose dolphin (*Tursiops truncatus*), false killer whale (*Pseudorca crassidens*), spotted dolphin (*Stenella frontalis*), striped dolphin (*Lagenorhynchus obliquidens*), killer whale (*Orincus orca*), and pilot whale (*Globicephala melaena*). For comparison, the *St* data of the slower, less efficient swimming beluga whale (*Delphinapterus leucas*; family Monodontidae) are included but treated separately.

The objective of this study was to investigate the range of *St* preferred by swimming cetaceans; how this range varied between species and within species; and most critically, what range of *St* numbers corresponded to maximum propulsive efficiency. Data directly relating swimming efficiency to *St*, for any swimming animal, have previously not existed [7].

RESULTS

A total of 267 swimming sequences for captive odontocete cetaceans was used for kinematic analysis (*S. frontalis*, n = 13; *L. obliquidens*, n = 17; *T. truncatus*, n = 107; *P. crassidens*, n = 69; *G. melaena*, n = 12; *O. orca*, n = 30; *D. leucas* = 19). The computed *St* showed little dependence on body length or swim speed. When aggregating animals for each species, the mean *St* values generally reside near the lower bound of the 0.25- to 0.35-range predicted for peak propulsive efficiency (Table 1). Excluding *D. leucas,* the mean *St* for all the delphinids was 0.26 ± 0.05 (n = 248). The predicted 0.25 to 0.35 *St* range captured 55% of the delphinid *St* data, (Figure 1), whereas the range from 0.2 to 0.3 contained 74%. For an incremental *St* range of 0.05, the majority of the data was found between 0.22 to 0.275 (44%).

TABLE 1. Mean Strouhal values.

Data Set	Odontocete Cetacean	Mean ± SD
n = 107	*Tursiops truncatus*	St = 0.26 ± 0.05
n = 69	*Pseudorca crassidens*	St = 0.26 ± 0.05
n = 30	*Orcinus orca*	St = 0.28 ± 0.05
n = 12	*Globicephala melaena*	St = 0.24 ± 0.02
n = 17	*Lagenorhynchus Obliquidens*	St = 0.24 ± 0.03
n = 13	*Stenella Plagiodon*	St = 0.34 ± 0.03
n = 19	*Delphinapteras leucas*	St = 0.35 ± 0.10

Propulsive efficiencies, which were previously reported by Fish [8], are plotted as a function of the corresponding *St* in Figure 2. For *P. crassidens*, *O. orca*, and *T. truncatus* propulsive efficiencies were found to broadly peak at about 0.90, 0.87, and 0.85 respectively, over a relatively narrow range of *St*, 0.23 to 0.28. Outside this *St* range, where measurements exist, efficiencies drop off rapidly. The *St* range favored by *P. crassidens*, *O. orca*, and *T. truncatus* was within this same range, 0.225 < St < 0.30 (Table 1). The efficiency data for *D. leucas* was lower, 0.83, and exhibits a conspicuously broader peak between *St* = 0.25 to 0.40. Furthermore, *D. leucas* did not appear to favor a particular *St* range. It has been noted that the swimming performance and efficiency of *D. leucas* differs considerably from other cetaceans, which is consistent with its general body contour and low-aspect-ratio flukes [8].

Strouhal numbers are often expressed as the product of nondimensional peak-to-peak fluke amplitude, A/L, and non-dimensional frequency, $f/(U/L)$. Regardless of St value, 89% of all (n = 267; including $D.$ $leucas$) the A/L data fall within a range of 0.15 to 0.25, and 90% of all (n = 267; including $D.$ $leucas$) the $f/(U/L)$ data fall within a range of 1 to 2. Note, if A/L = 0.15 to 0.25 and $f/(U/L)$ = 1 to 2, the corresponding St range is 0.15 to 0.50. Presumably the "boundaries" imposed by A/L and $f/(U/L)$ on St are manifestations of additional morphological and hydrodynamic constraints imposed on the animal.

FIGURE 1. Histogram of percentage of total odontocete cetacean Strouhal Number data (n = 248).

CONCLUSIONS

The St data presented in this study significantly expand the previous data set for cetaceans by greatly increasing the number of observations (more than 100-fold), species (6-fold), range of Reynolds number (10-fold), and for the first time, provide a direct comparison between measured St number and swimming efficiency. The present data show that over the range of swim speeds observed, 2 to 8 m s^{-1}, cetaceans swim at St values between 0.2 and 0.4, preferring a range of 0.2 to 0.3 where maximum efficiencies occur. Although slightly lower than predicted, the strong relationship between maximum propulsive efficiency and St found in the present data continues to support the premise [3, 4, 5] that vorticity control associated with fluke kinematics is an important attribute of cetacean swimming performance. Ultimately, it is hoped that a better understanding of what nature has evolved for efficient, underwater, propulsive motion will provide new strategies toward improving the efficiency and performance of underwater vehicles.

FIGURE 2. Propulsion efficiency of cetaceans as a function of Strouhal Number (St). Colors indicate particular species: *Tursiops truncatus* (red), *Pseudorca crassidens* (green), *Orincus orca* (blue), *Delphinapterus leucas* (black).

REFERENCES

1. Rosen, M. W. 1959. "Water Flow about a Swimming Fish," *U.S. Naval Ordnance Test Station Technical Publication* 2298, pp. 1–96.
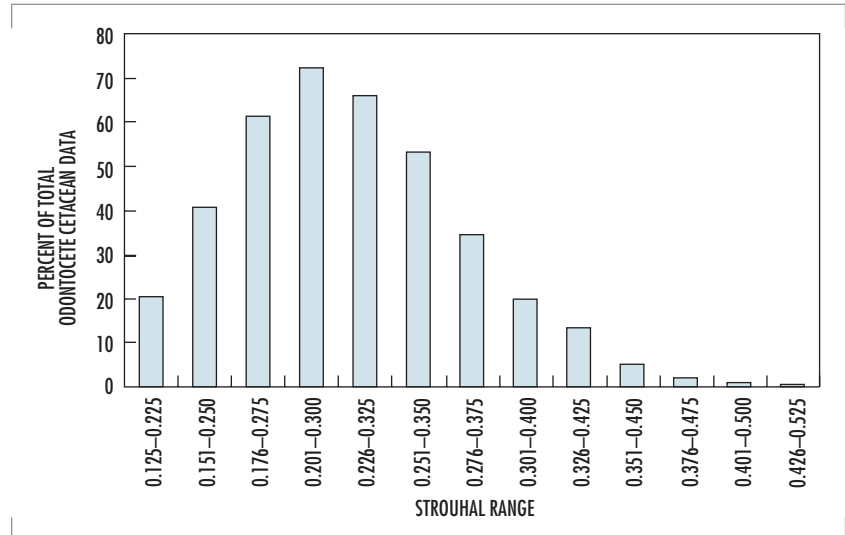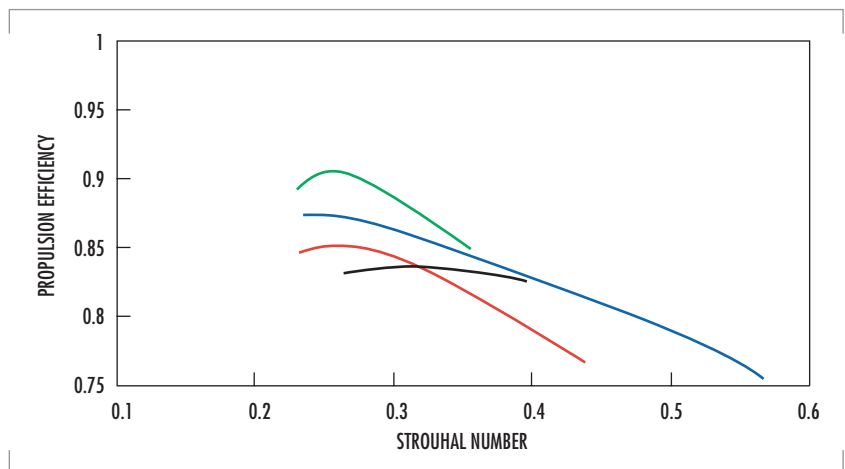
2. Rosen, M. W. 1961. "Experiments with Swimming Fish and Dolphins," *Publication 61-WA-203*, American Society of Mechanical Engineers, pp. 1–11.

3. Triantafyllou, M. S., G. S. Triantafyllou, and R. Gopalkrishnan. 1991. "Wake Mechanics for Thrust Generation in Oscillating Foils," *Physics of Fluids*, vol. 3, pp. 2835–2837.

4. Triantafyllou, G. S., M. S. Triantafyllou, and M. A. Grosenbaugh. 1993. "Optimal Thrust Development in Oscillating Foils with Applications to Fish Propulsion," *Journal of Fluids and Structures*, vol. 7, pp. 205–224.

5. Triantafyllou, G. S. and M. S. Triantafyllou. 1995. "An Efficient Swimming Machine," *Scientific American*. vol. 272, pp. 40–48.

6. Lang , T. G. and D. A. Daybell. 1963. "Porpoise Performance Tests in a Seawater Tank," *U.S. Naval Ordnance Test Station Technical Report*, vol. 3063, pp. 1–50.

7. Bandyopadhyay, P. R. 2002. "Maneuvering Hydrodynamics of Fish and Small Underwater Vehicles," Integrative and Comparative Biology. vol. 42, pp. 102–117.

8. Fish, F. E. 1998. "Biomechanical Perspective on the Origin of Cetacean Flukes," *The Emergence of Whales: Evolutionary Patterns in the Origin of Cetacea,* (ed., J. G. M. Thewissen), Plenum. New York, NY, pp. 303–324.
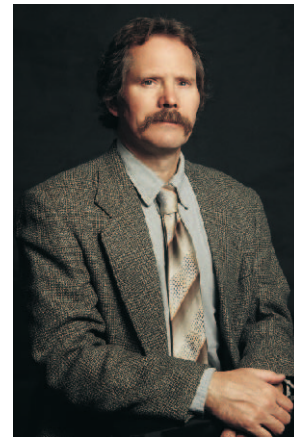
Jim Rohr

Ph.D., Engineering Physics, University of California, San Diego, 1985

Current Research: Flow-stimulated bioluminescence; dolphin swimming.

Frank Fish

Ph.D., Zoology, Michigan State University, 1980

Current Research: Energetics and hydrodynamics of aquatic locomotion by vertebrate animals and the application of biological designs to biomimetic engineered systems.