Optimal Mating Strategies for Simultaneous Hermaphrodites in the Presence of Predators

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Two *Physa acuta* snails

Introduction:

Certain preferentially outcrossing simultaneous hermaphrodites are capable of self-fertilization. While self-reproducing organisms transmit two sets of genes to offspring, often inbreeding depression leaves outcrossing the better strategy. The accepted model to optimize self-reproductive delay fails to account for certain organisms' response to predators. In this poster, we discuss the theoretical implications of incorporating this response, including life history scenarios and the existence of hysteresis, the lack of reversibility as a parameter is varied.

Model:

In previous research, Tsitrone et al. constructed a mathematical model for optimal mating strategies in simultaneous hermaphrodites, using *Physa acuta* for testing.[1,2] We have exchanged Tsitrone's constant mortality rate for a linear piecewise function, with a few related additional changes to the rest of the model, resulting in the following:

Average Number of Offspring in a Lifetime:

$$R_0(\tau) = \int_0^\tau \left(\int_u^\infty l(x,u) b_o(x,u) dx \right) f(u) du + \int_\tau^\infty \left(\int_\tau^u l(x,\tau) b_s(x,\tau) dx + \int_u^\infty l(x,\tau) b_o(x,\tau) dx \right) f(u) du.$$

Instantaneous Rate of Gene Transmission:

Outcrossing:

$$b_o(x,t) = \begin{cases} 0, & x < t \\ c(1+k_r r t), & x \geq t \end{cases}$$

Selfing:

$$b_s(x,t) = \begin{cases} 0, & x < t \\ 2(1-\delta)c(1+k_r r t), & x \geq t \end{cases}$$

Mate Encounter Distribution:

$$f(u) = e_m \exp(-e_m u)$$

Survival Probability Distribution:

$$l(x,t) = \exp\left[-\int_0^x m(v,t) dv \right]$$

Mortality:

$$m(x,t) = \begin{cases} m_0 + \epsilon e_p(1 - k_d(1-r)x - k_s r x), & x \leq \min(t,\tau_c) \\ m_0 + \epsilon e_p(1 - k_d(1-r)t - k_s r t), & x > \min(t,\tau_c) \end{cases}$$

Parameters:

c:	Baseline Reproduction	e_m:	Mate Encounter Rate
m_0:	Baseline Mortality	e_p:	Predator Encounter Rate
k_r:	Resource Allocation Efficiency	ϵ:	Predator Success Rate
k_s:	Size Defense Efficiency	δ:	Inbreeding Depression
k_d:	Other Defense Efficiency	r:	Resources Allocated to Growth

Model Predictions:

Our recent work on the model has been focused on ***r***, the fraction of resources invested in size. While other parameters, such as predation rate or mate encounter rate, represent environmental factors, ***r*** represents various behavioral and physiological attributes of the organisms themselves, and, as such, can be selected for and optimized. Unfortunately, as our outer integrals cannot be analytically solved, we cannot analytically optimize ***r***, and must instead use numerical optimization.

Below we present the results ***r*** sweeps in different environmental scenarios. e_p and k_s are the two parameters that vary from sweep to sweep, while the remaining columns are various values of interest: r^* and τ^* are the optimum ***r***- and ***τ***- value, respectively; R_0 is fitness at r^* and τ^*. The remaining two values are derived from r^* and τ^*. ($r^* \bullet \tau^*$) represents comparative "size", and likewise ($(1-r^*)\tau^*$) "defense".

Scenario	e_p	k_s	k_d	r^*	τ^*	R_0	$r^* \bullet \tau^*$	$(1-r^*)\tau^*$
No Predator	0	0.1	0	1.0	30.0	400.4	30.0	0.0
Newt	3/7	0.2	0	1.0	30.0	101.4	30.0	0.0
Fish	3/7	-0.2	0.2	0	5.0	60.0	0.0	5.0

In each of the above scenarios:
$c = 18$, $m_0 = 0.05$, $\epsilon = 1$, $e_m = 0.1$, $k_r = 0.1$, $\delta = 0.8$

Another prediction of interest is in organism's investment strategy. As discussed above, ***r*** is a parameter that can be optimized by natural selection, and as such, we present some sweeps of how optimum ***r*** values change as the ratio of the defensive parameters, k_s / k_d, changes.

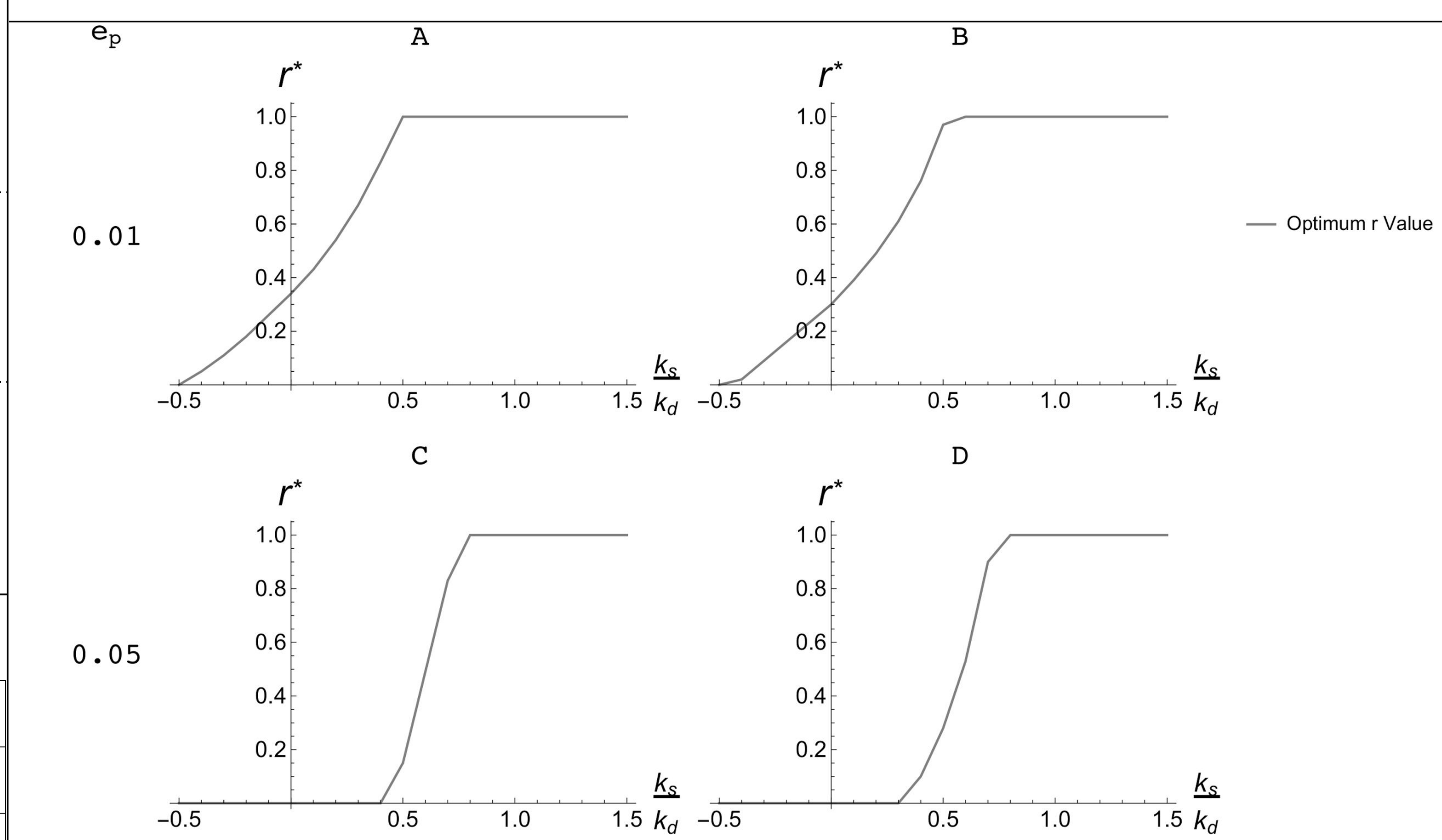

Figure 1.

In the left hand column:
$c = 18$, $m_0 = 0.05$, $\epsilon = 1$, $k_d = 0.1$, $e_m = 0.1$, $k_r = 0.1$, $\delta = 0.8$

In the right hand column:
$c = 18$, $m_0 = 0.05$, $\epsilon = 1$, $k_d = 0.1$, $e_m = 0.01$, $k_r = 0.1$, $\delta = 0.75$

Experimental Results:

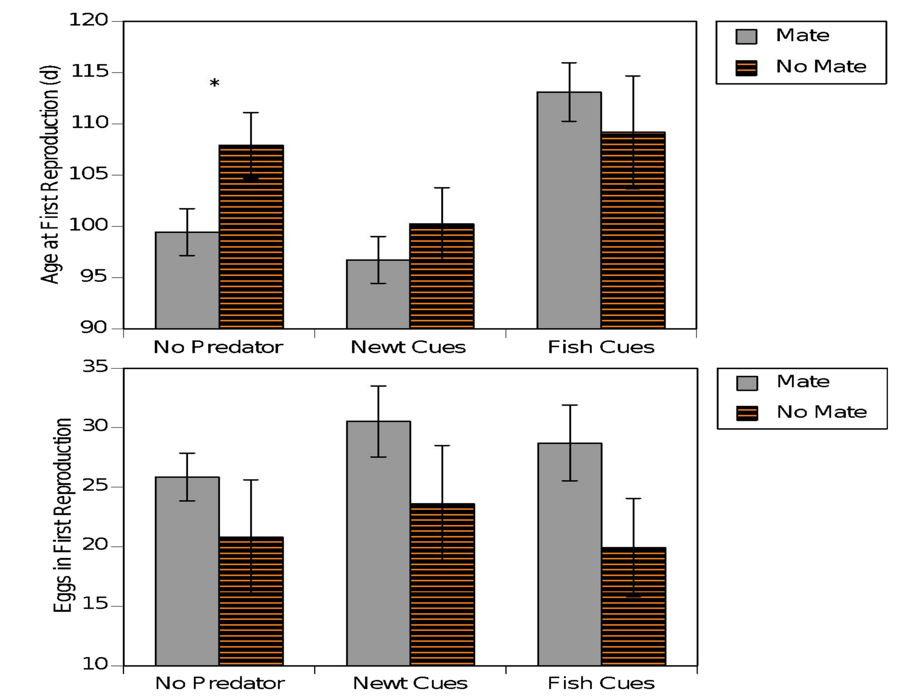

Figure 2a. The top bar graph is age at first reproduction for *Physa acuta* snails reared in three different predator-cue treatments and with/without access to mates. The bottom bar graph depicts eggs laid at first reproduction by *Physa acuta* snails reared in three different predator-cue treatments and with/without access to mates.

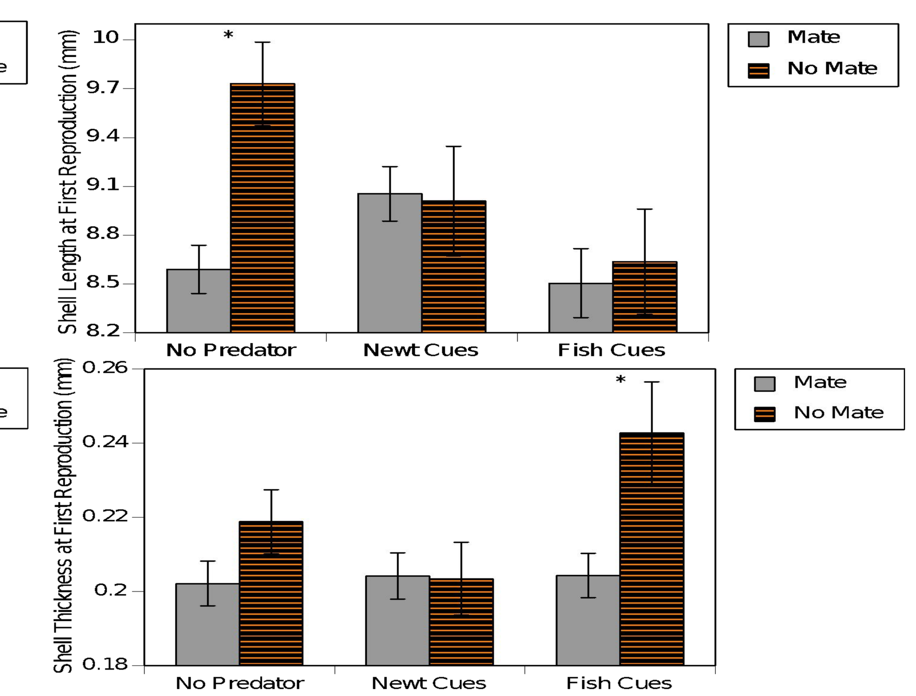

Figure 2b. The top bar graph depicts the shell length at first reproduction for *Physa acuta* snails reared in three different predator-cue treatments and with/without access to mates. The bottom bar graph illustrates shell thickness at first reproduction for *Physa acuta* snails reared in three different predator-cue treatments and with/without access to mates.

In addition to our analytical work, we have recently completed some experimental model validation, some results of which are displayed above. The comparison between our model and the experimental results is mixed, and we've no analogue for fitness. By parts:

Age at first reproduction: Our model predicts τ, which, in the above results, is the difference between the No Mate and Mate columns. There is no statistical significance between the two in the Newt and Fish cases, which is not terribly out of place for the Fish, but significantly different than our predictions for the Newt. There is a significant delay in the No Predator case, which is expected.

Shell Length: Our model predicts size to be roughly equal in the No Predator and Newt cases, with the Fish case to be significantly less. Averaging between the Mate and No Mate cases in each of these scenarios, this correlates with the experimental data.

Shell Thickness: As shell thickness corresponds to the snails' main defense mechanism–other than size–our model predicts the No Predator and Newt cases to have the same thickness, and the the Fish case to have some defense built up. This is reflected in the data.

Acknowledgments

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Literature Cited

1. Tsitrone, A., Duperron, A., & David, P. 2003. Delayed selfing as an optimal mating strategy in preferentially outcrossing species: theoretical analysis of the optimal age at first reproduction in relation to mate availability. American Naturalist, 162, 318–331.
2. Tsitrone, A., Jarne, P., & David, P. 2003. Delayed selfing and resource reallocations in relation to mate availability in the freshwater snail *Physa acuta*. American Naturalist, 162, 474–488.