McKibben Webster Chapter 11 – Partial Solutions and Hints

MATLAB Exercise 11.1.1

i) a) For the GUI of the homogenous case, see the MATHLAB Exercise 9.1.2. While both of these cases start out with a sine curve, the homogenous case slowly degrades into a constant function. However, the non-homogenous case with the first forcing function becomes, essentially, a sine graph from 0 to π while passing through a function that is nearly constant in the left half and a parabola-like peak in the right half. Thus, while the homogenous case becomes uniform throughout, the forcing function f1, which is a constant over x and t, causes the object to retain heat in the center, and be neutral at the edges. Intuitively, this makes sense: heat bleeds quicker from the edges, and the middle will heat towards the edges, causing a smooth curve with the center being the peak.

b) In chapter 9, a higher k^2 value causes the function to reach the end solution "quicker" than a lower value. In essence, a k^2 value of 1 took until about t=0.6 to reach a straight line across at 0. However, a k^2 value of 2 was practically a straight line at around t=0.33. Therefore, one possible assumption is to expected that a higher k^2 value will cause a quicker evolution to a constant, uniform heat. In actuality, there are two results. First, the higher k^2 value does cause the function to settle quicker; however, the function settles at a lower value. So, the k^2 value both increases the speed of evolution and lowers the heat across the object. This also makes sense: faster heat diffusion naturally would imply a lower heat across the object.

c) Based on the above discussion, we would expect a slower evolution, but a higher end heat level across the entire object. The GUI does reflect this. Although the final sine-like curve has not yet been reached at t=0.5 (thus showing the slower evolution of the function), the max heat is significantly higher than in the similar position for k^2 =1.

d) The evolution of this function is practically identical to the case with A=1, however, the maximum of the function is reduced by about one-half. Thus, this function is approximately halfway between the A=1 case and the homogenous case at t=0.5.

e) Examination shows that the end solution does seem to converge to the homogenous case.

f) Larger values of A should result in higher peaks in the resultant sine-like end curve.

g) In essence, the same conjectures all hold. The only primary difference is the initial evolution of the function, but the end result (a sine-like curve) is still the same.

h) Same as part g).

ii) Using the fifth forcing function does not change the results much, if any. All of the different conjectures made in part i) still hold, and the end function still appears sine-like.

iii) Note that this is using a Neumann boundary condition.

a) While the homogenous case ended with a straight line at 1, this non-homogenous case evolves to a negative cosine-like graph, sloping upward from x=-1, passing through 1 at about x=0.4.

b) As seen in the above conditions, higher k^2 values cause a faster evolution of the function. This is confirmed by the function. However, as with the previous examples, note that the "amplitude" of the function is lower. The left side of the graph reaches around 0.7, while before it reached 0.6. The same is true of the right-hand side as well. In other words, the heat is more uniform across the object modeled.

c) As before, the lower k^2 value slows the evolution, and also makes different areas more extreme than in higher values. Note the left side is around 0.5, and the right around 1.2.

d) This solution lies between the homogenous and the one from part a).

e) These do indeed converge to the homogenous case.

f) For this, the left side of the graph gets more negative, and the right side becomes more positive. However, given enough time, the right side as well will be overpowered by the Bt term, and start to drop.

g) Similar to the previous initial condition, the solution evolves into a graph looking like a negative cosine, while the homogenous evolved into a straight line at around 0.09. Given the same conjectures made for the previous initial condition, there are no contradictory data from the GUI.

h) Again, this solution evolves into a negative-cosine-like graph. The homogenous evolved into a straight line around 0.5. This solution also has no contradictions based on the conjectures from the previous examples.

iv) Again, this uses the Neumann boundary condition.

a) The homogenous equation evolves to a straight line at 1. This equation evolves quickly into a line that moves up that graph. By t=0.5, the line is at about 1.4.

b) It would be expected that a higher k^2 value would cause the graph to move faster, settle faster, or something similar. This is seen in all the previous examples. However, here, the k^2 value does not seem to effect the end result at all. However, the cosine curve does end quicker than in the $k^2=1$ case. This is seen if you reorient the view of the graph to viewing from the side (t at the bottom, z on the left, and x straight out of the screen).

c) Based on the above conclusion, you should expect the graph to take longer to settle into the straight line. This is correct, but once again the end line is unchanged. Again, viewing the graph from the side is the best way to see this.

d) Here, the end line is at a larger value; in other words, this lower value of *a* increases the end line.

e) As *a* gets smaller, the line rises higher. So, no, the solution does not approach the homogenous case as *a* approaches 0.

f) On the other hand, note that as *a* gets larger, the line gets lower, and approaches a line at 1. So, in fact, the solution approaches the homogenous case as *a* approaches infinity.

g) The homogenous case straightens to a line at about 0.09. The non-homogenous case straightens to a line that keeps rising, and at t=0.5 is at about 4.8. When changing k^2, again the primary alteration is the speed at which the function evolves into a line. The end line is unaltered. Also, the same results as the previous example are seen when altering *a*.

h) The homogenous case ends with a line at about 0.5. This non-homogenous case ends with a line at about .88. The same conjectures are once again valid for both changing k^2 and *a*.

v)

a) The homogenous case rapidly evolves to a line at 1. This non-homogenous case evolves to a line that gradually rises.

b) As with other k^2 values, this higher value causes a faster stabilization into the line. To see this easier, view the graph from the side (t bottom, z on the left, x coming out of the screen).

c) As expected, a lower k^2 value slows the stabilization into the line.

d) This graph appears to be the same as the one in part a).

e) These too appear to be the same. These do not, by any means, converge to the homogenous solution. Instead, they seem to be converging to the forcing function f1, the constant forcing function.

f) Using a value for ω to be 20, it is plain to see the oscillations. If you examine the values of ω from parts c) and d) again, but for a longer time period, you can see that they will oscillate at different speeds. They just appear the same because for just t=0.5, they are close to constant (i.e., cos(ωt) is still close to 1). This is why when ω=> 0, the forcing function appears to converge to f1. On the other hand, greater ω values cause faster oscillations.

g) There is no real difference except the initial start. The end result is a line which will oscillate at a speed determined by ω.

h) There is no real difference except the initial start. The end result is a line which will oscillate at a speed determined by ω.

MATLAB Exercise 11.1.2

i)

a) By t=2, the solution is steady at a smooth sine-like curve peaking at about 0.25. The homogenous case is just a straight line at 0.

b) This solution has a lower peak than the one in part a). Its peak is about 0.1. This is thus about halfway between the solution in part a) and the homogenous solution.

c) As A goes to 0, the function approaches the homogenous case. The long term solution of the system is determined by A, where higher values yield a higher peak to the sine-like curve, and vice-versa.

d) There are no real differences in the solutions.

e) There are no real differences in the solutions.

ii)

a) By t=2, the solution is again a sine-like curve with a peak at around 0.2. The homogenous case is just a straight line at 0.

b) The function peaks lower than the original in part a). It is partway between the part a) solution and the homogenous case.

c) As A goes to 0, the function approaches the homogenous case. The long term solution of the system is determined by A, where higher values yield a higher peak to the sine-like curve, and vice-versa.

d) There are no real differences in the solutions.

e) There are no real differences in the solutions.

iii)

a) In the homogenous case, the solution goes to a line at 1. However, this non-homogenous case will keep decreasing. The left side is always lower than the right, but the whole curve as a whole gradually decreases.

b) With the lower values of the constants, the function does not decrease as fast as with the higher value. So, in fact, this is closer to the homogenous case then part a).

c) Basically, the higher A and B are, the faster the function decreases. Smaller values yield a smaller drop, and thus the solution converges to the homogenous case as A and B converge to 0.

d) There are no real differences in the solutions.

e) There are no real differences in the solutions.

iv)

a) In the homogenous case, the solution goes to a line at 1. In this solution, the function approaches a straight line at 2.

b) With A=0.5, the graph now approaches a line at 3. This is more extreme than the part a) solution.

c) Smaller values of A causes the end line to be at a higher value. Conversely, larger values cause the end line to be closer to the homogenous solution of 1.

d) There are no real differences in the solutions.

e) There are no real differences in the solutions.

v)

a) In the homogenous case, the solution goes to a line at 1. This solution oscillates in a straight line from 2 to 0 in a sinusoidal fashion.

b) Decreasing the value of ω does two things. First, it slows the oscillations. Second, it increases the amplitude inversely proportional to the factor ω was decreased by. Note that here the oscillations are from -1 to 3. Compared to the homogenous case, this is farther from the line then part a).

c) Increasing the value of ω quickens the oscillations and decreases the amplitude. Thus, as ω increases to infinity, the solution approaches the homogenous solution.

d) The graphs are practically the same excepting the starting function. All, however, quickly evolve into an oscillating line, the frequency and amplitude defined by ω as mentioned above.

e) The graphs are practically the same excepting the starting function. All, however, quickly evolve into an oscillating line, the frequency and amplitude defined by ω as mentioned above.

MATLAB Exercise 11.1.3

i)

a) When using the SUP option, both sides lag behind the original graph slightly. They catch up at around t=0.2, with the left side catching up slightly before the right side. The L2 and the ALL options are practically the same as the SUP option. After about t=0.25, all three options end up slightly greater than the original.

b) When k^2 was less than 2, the modified function was delayed initially, and then was slightly greater than the function at the end. When k^2 is greater than 2, the modified function was ahead of the original function, but ended up lower at the end. In both cases, the closer the new k^2 was to the original, the closer the graphs were. Thus, the graph does vary continuously with respect to k^2.

c) Here, the only difference from A=0 is the graph reaches a higher end point and moves faster. With the different k^2 values, the difference from the original to the modified version is more pronounced, but still follows the same pattern as from (a) and (b).

d) Similarly, the lower A value means a smaller end point and thus moves slightly slower in the last part of the solution. The difference from the modified version to the original is thus slightly less, but still follows the same pattern.

e) As mentioned above, the stronger forcing function creates a greater difference, and a greater increase in difference later in the solution. It also shifts the time where the two norms are the same.

f) There are no significant changes in the solution with the different starting function. All of the conjectures still hold.

g) There are no significant changes in the solution with the different starting function. All of the conjectures still hold.

ii) Overall, there is very little difference in the solutions from the first forcing function (part i) and the fifth forcing function. f1 is constant and non-zero over the entire interval, and f5 is constant and non-zero over a portion of it, and zero elsewhere. The primary difference lies in the end of the solution, where the first forcing function is slightly higher than the fifth, keeping all other constants the same.

a) When using the SUP option, both sides lag behind the original graph slightly. They catch up at around t=0.2, with the left side catching up slightly before the right side. The L2 and the ALL options are practically the same as the SUP option. After about t=0.25, all three options end up slightly greater than the original.

b) When k^2 was less than 2, the modified function was delayed initially, and then was slightly greater than the function at the end. When k^2 is greater than 2, the modified function was ahead of the original function, but ended up lower at the end. In both cases, the closer the new k^2 was to the original, the closer the graphs were. Thus, the graph does vary continuously with respect to k^2.

c) Here, the only difference from A=0 is the graph reaches a higher end point and moves faster. With the different k^2 values, the difference from the original to the modified version is more pronounced, but still follows the same pattern as from (a) and (b).

d) Similarly, the lower A value means a smaller end point and thus moves slightly slower in the last part of the solution. The difference from the modified version to the original is thus slightly less, but still follows the same pattern.

e) As mentioned above, the stronger forcing function creates a greater difference, and a greater increase in difference later in the solution. It also shifts the time where the two norms are the same.

f) There are no significant changes in the solution with the different starting function. All of the conjectures still hold.

g) There are no significant changes in the solution with the different starting function. All of the conjectures still hold.

iii) Note this is a Neumann boundary condition.

a) In both norms, the modified graph becomes exactly the original graph, shifted downwards. At the start, the differences make a cosine graph shifted down from the original, and follow it exactly. For the sup norm, the difference is 0.1, and for the L2 norm, it is about 0.142. Other than that difference, they are exactly the same.

b) In all the graphs, the modified version is shifted downwards a number of units. With the sup norm, it is exactly the perturbation value. With the L2 norm, it is about 40% more of the perturbation value.

c) The difference downwards is still the same as in parts (a) and (b): the perturbation value for the sup norm, and 40% more than the perturbation value for the L2 norm.

d) The difference downwards is still the same as in parts (a) and (b): the perturbation value for the sup norm, and 40% more than the perturbation value for the L2 norm.

e) Here, the forcing function did not change the effect the perturbation value had on the solution.

f) With the different initial condition, the shift downward is much less. Instead of the being about the same as the perturbation value for the sup norm, and 40% of that value with the L2 norm, the sup norm is a shift downwards of about 10% of the perturbation value, and the L2 norm of about 12% of the perturbation value. This holds for all the different perturbation values. Altering the forcing function does not change this shift. The shift just might appear smaller due to the larger change in the solution.

g) Using a step function initial condition, the modified version is again shifted downwards from the original. At the beginning, the sides are shifted only slightly, and the middle is shifted a larger amount. However, the difference gradually evens out, and the difference averages to about 50% of the perturbation for the sup norm and about 72% for the L2 norm. Again as above, altering the forcing function does not change the effect the perturbation values have on the new function. It is still a shift downward of 50% or 72% of the perturbation value, depending on which norm is used.

(iv)

a) The solutions quickly evolve into parallel lines gradually shifting upwards. The modified versions are shifted downwards 0.1 units for the sup norm and 0.142 units for the L2 norm. Beyond that, they are the same.

b) There is very little difference in the solutions except as noted above. The modified graph is shifted downwards a number of units equal to the perturbation value when using the sup norm, and about 42% more than the value for the L2 norm. The k^2 value only alters the speed at which the cosine graph evolves to a straight line. This is hard to see, as generally it only takes under 5 frames to evolve, but to see it better, try extremely high or extremely low perturbed values of k^2. Thus, the k^2 value only slightly affects the graph (and thus the graphs are continuous over them, being nearly constant) while the perturbation value continuously affects the graph with a net shift downwards directly proportional to the perturbation value.

c) Using the higher *a* value in the forcing function only affects the final resting point of the solution. It does not affect the evolution speed of the graph to a straight line (ie, the k^2) or the downward shift (the perturbation value).

d) Similarly, a lower *a* value has no affect on the evolution speed nor the downward shift.

e) As mentioned above and easily seen in the graphs, there is no change caused by the new forcing function.

f) When utilizing the exponential initial condition, there appears a strange bobble in the graph of the differences. The greater the difference in k^2 values and perturbation value, the larger the bobble. This bobble occurs as the modified graph is greater than the original in some areas and less in others, which stems from the different k^2 values. That is the only difference in the solutions and appears at about t=0.75, depending on the size of the bobble. Otherwise, the k^2 values simply affect how quickly the graph evolves to the line, and the perturbation value is a downward shift of the perturbation value for the sup norm and about 40% more than that value for the L2 norm.

g) In this solution, there is again a strange bobble around t=0.1. However, as k^2 gets closer to the original value and the perturbation values decrease, the bobble straightens out a shifts slightly to a later time. It appears more prominently in the sup norm, but the L2 norm also has it to a lesser degree. This bobble occurs as the modified graph is greater than the original in some areas and less in others, which stems from the different k^2 values. However, the end solutions of the graphs are defined the same as above, with the sup norm shifted downwards 50% of the value, and the L2 norm shifted downwards 72% of the value. Again, the forcing function does not cause any changes.

(v)

a) The two norms are practically identical except for a difference in how shifted each is. Beyond that, the lower k^2 value means the graph evolves to a straight line slower. The different ω value causes the graphs to slowly diverge from the difference of the perturbation value (for the sup norm) and about 1.4 the perturbation value (for the L2 norm).

b) So here, the k^2 value alters the speed of the evolution of the solution into the straight line. The perturbation value controls the downward shift, varying also with the norm used. The ω value controls the speed of the long term oscillations of the line. However, up to t=0.5, it really only slowly changes the downward shift over time. Higher ω's cause the shift to increase from the original shift, lower to decrease. Thus, all of the variables vary the solution continuously.

c) The solutions with the exponential initial condition are very similar to the cosine solutions. The end result is the same, with parallel lines diverging or converging with respect to time, dependent on the different ω values. The k^2 value again controls the speed of evolution to that line and the size of the bobble (mentioned in (iv) f), while the perturbation controls the size of the base downward shift.

d) The solutions with the exponential initial condition are very similar to the cosine solutions. The end result is the same, with parallel lines diverging or converging with respect to time, dependent on the different ω values. The k^2 value again controls the speed of evolution to that line and the size of the bobble (mentioned in (iv) g), while the perturbation controls the size of the base downward shift.

(vi)

MATHLAB Exercise 11.1.4

(i) Note this is a Dirchlet IBVP.

a) In the homogenous case, the curve is centered at 0, and (eventually) curves upwards. However, with the non-homogenous, the curve centers at 1, the BC conditions. The curve does eventually reach a steady solution, with it curving upwards from 1, hitting a peak at about 1.22 when x=0, and then curving back down to 1. The function rather quickly moves from the initial state into the above curve, with the majority of the time elapsed spent reaching the peak point.

b) The two solutions basically parallel each other, 1 unit apart. The homogenous curve is based at 0, and the non-homogenous at 1, but they both extend about 0.22 units upwards from their base. Their initial evolution is different, but their end result is the same, just shifted upwards or downwards.

c) Whatever your choice of T1 and T2 are, the result will be a curve ending at those values and rising upward from the line between them. The graph also will tend to bulge higher near the higher of the two endpoints.

d) Increasing A just increased the amplitude of the end curve.

e) Similarly, decreasing A decreased the amplitude of the end curve.

f) Using the exponential initial condition only changes the initial behavior, not the end result. The curve centers on the line from the left to right endpoints and curves upward an amount proportional to A. However, when A=0.01, the end curve has not quite reached an upward curve, instead, it is still approaching a straight line at 1 from below. This is because the initial condition is below that line, and there has not been enough time for it to reach that point.

g) Similarly, the step function initial condition also only changes the initial behavior, with the exception of when A=0.01. There, once again, it still is approaching the horizontal from below.

h) The changing initial conditions did not alter the end function by much, except as mentioned in the A=0.01 case above. The primary alteration was in the initial behavior, but the function quickly evolved into an upward arcing curve centered at the line between the two boundary conditions. However, when A is low, the function evolves slower from the initial conditions, altering that end behavior.

ii) Utilizing the fifth forcing function only alters the behavior of the function in one respect: the center of the graph tends to bulge upwards, and the sides lag behind the equivalent graph in part 1. This makes sense, as the step function is basically a limited version of the constant function from part i. All other characteristics of the graph stay similar with that one alteration.

iii) This is the Neumann BC.

a) The graph rapidly evolves from the cosine initial condition into a nearly straight line from 1.6 on the left to about 0.3 on the right by t=0.3. After that point, the graph gradually shifts downwards, ending with the left side being at about 1.3 and the right at -0.3. The graph does not evolve to a steady-state temperature, instead, the temperature gradually lowers across the entire interval and continues dropping while maintaining the negative-sloped line.

b) While the non-homogeneous graph rotates to the right (giving a negative slope), the homogeneous graph rotates left, and eventually becomes a curve reminiscent of an Arctan graph.. However, the graphs then proceed to lower in parallel, and both to not attain a steady state solution.

c) When changing the left condition, making the left condition positive drops the left boundary. In other words, the left boundary is lower than the right boundary the more positive it is. On the other hand, making the left boundary more negative increases the left boundary. A left boundary of 0 keeps the left boundary around 1 initially. Similarly, lowering the right BC condition lowers the right boundary, raising it raises it, and setting it equal to 0 keeps the right boundary around 1 initially. Thus, there are nine possible combinations: (+,+) which curves upward from left to right , (+,0) which curves up gently, but the right-hand boundary does not go up very far, (+,-) which is similar to a downward opening parabola with the vertex slightly shifted to the right (depending on the exact values), etc. If both are negative or positive, it is a smooth curve from left to right, either increasing or decreasing. If one if negative and the other is positive, the graph is similar to a parabola. If one is a 0, the graph is stays close to 1 on that side, while the other side is dictated by whether it is positive or negative. If both are 0, then that is the homogeneous case. In all cases, the graph gradually starts shifting downwards as a whole after the curve of the graph stabilizes.

d) With the higher A value, the left side drops faster initially (Ax is negative on the left side) and the right side rises faster initially. The result in the case of -1,-1 BC conditions is reminiscent of a negative cubic with one local min and one local max. Including any alterations added with different boundary conditions, the left side drops quicker than the original values, while the right side rises quicker initially. Eventually, the graph stabilizes, and the graph shifts downwards as a whole, as in the above. Thus, unlike with the A and B values above, the graph does not smooth out. Instead, it contains a downward bulge on the left and an upward bulge on the right compared to the first graphs.

e) Reducing the A and B values to nearly 0 does two things. First, reducing A to 0 straightens out the end graph. As seen in the previous part, the higher A value causes bulges in the graph. Thus, the lower A value straightens the graph out. This included when the boundary conditions are the opposite sign, the result is nearly a perfect parabola when the boundary conditions have the same magnitude. Reducing B to nearly 0 eliminates the end downward shifting. Instead, the graph straightens and then stays still.

f) With the exponential initial condition, there are no great changes to the overall behavior of the solution. The initial condition only changes the initial stages of the graph, not the overall behavior. The graph quickly evolves into the forms seen and described with the cosine initial condition.

g) Similarly, the step function initial condition only changed the initial stages; the graph quickly evolved into the forms described with the cosine initial condition.

h) In summary, the initial conditions did not alter the end solution very much. All that changed wass the speed at which the graph evolved into the normal solution. The cosine graph would quickly evolve into the normal shape, while the step function graph also would quickly evolve. The exponential function evolved the slowest, but still the result quickly settled into the solutions seen in the other two initial conditions. In all three cases, changing the parameters of the forcing function had the same effect: A altered the "smoothness" of the graph, higher values yielding more bulges while lower values yield a straighter curve. The B value, in turn, altered the speed of the ending downward shift.

iv)

a) The cosine initial condition quickly disappears as the solution evolves into a straight line from about 2 on the left to 0 on the right by about t=0.45. It is hard to determine this from plotting until t=1, but the graph now slowly oscillates up and down as that line. Plotting until t=2 shows two peaks (one in the midst of the evolution from the initial conditions) and 1 full trough and part of the second. Thus, the solution does not reach a steady state temperature, as it is constantly rising and falling.

b) The homogenous case is a straight horizontal line that oscillates up and down. The homogeneous and non-homogeneous case oscillate in sync, and they also evolve to their respective lines at about the same rate.

c) Altering the boundary condition values have the same effect that appeared in part iii). Thus, when changing the left condition, making the left condition positive drops the left boundary. On the other hand, making the left boundary more negative increases the left boundary. A left boundary of 0 keeps the left boundary around 1 initially. Similarly, lowering the right BC condition lowers the right boundary, raising it raises it, and setting it equal to 0 keeps the right boundary around 1 initially. If the two boundary conditions have the same sign, one boundary is high and the other is low. The graph follows a nearly straight line between the boundaries, depending on the values of the conditions. If the boundary conditions have the same magnitude, it is a straight line. Otherwise, the graph is slightly curved. If the boundary conditions have opposite signs, the graph forms a parabola-like curve between the boundary conditions.

d) Increasing ω has two effects. First, it increases the speed of the oscillations. Secondly, it lessens the amplitude of the oscillations.

e) When using ω=0, the graph evolves into its line or curve, and then moves upwards. This makes sense, as the forcing just becomes a constant when ω=0.

f) The results utilizing the exponential initial condition are the same as using the cosine initial condition. The graph very quickly evolves into the oscillating graph as in the cosine initial condition. However, the max boundaries for the exponential graph are lower than that of the cosine graph. However, all the effects of altering

g) Similar to the exponential graph, the max boundaries are lower than that of cosine, but are larger than that of the exponential graph.

h) The overall evolution of the solution was not changed by the initial conditions. However, the maximum amplitude of the oscillations were changed. Cosine initial condition had the highest amplitude, followed by the step function, and finally the exponential graph. Altering the forcing function parameters did have the same influence in all situations.

MATHLAB Exercise 11.1.5

(i) Note this is a Dirchlet IBVP.

a) The graph starts out with peaks in two diagonal corners and troughs in the other two. The graph quickly evens out and ends with a single, centered peak. The graph evolves in a very similar way to the 1D graph, just expanded into two dimensions. thus, the one peak and one trough becomes 2 peaks and 2 troughs, in diagonal corners within the boundaries.

b) With a higher k^2, the graph evolves quicker than the original. Also, the end peak is lower due to the faster diffusion of the heat.

c) With the lower k^2, the graph evolves slower initially, but has a higher peak at the end.

d) The lower A value lowers the end peak of the graph.

e) As the value of A gets lower, the end peak of the graph gets smaller, approaching the plane. Thus, as A approached zero, the graph does approach the homogenous solution.

f) As A gets larger, the end peak gets higher and higher, and thus the graph evolves faster as well.

g) The conclusions for the exponential initial condition is the same as for the sine initial condition.

h) Again, the same end result occurs with the step function initial condition.

(ii) Using the fifth forcing function yields similar results as with the constant forcing function in part (i). With higher k^2 values, the graph evolves faster, and lower values make the graph evolve slower. The A values again effect especially the end peak, with high values meaning a higher peak, and low values giving a lower peak.

(iii) This is the Neumann boundary conditions.

(a) The graph quickly evolves from a central peak into a sloped plane, which is similar to the 1D equation, where it evolved from a central peak into a sloped line.

(b) As should be expected, the higher k^2 value makes the graph evolve quicker. The higher k^2 also lowers the slope of the end graph.

(c) Similarly, the lower k^2 value makes the graph evolve slower, but makes the slope of the end graph steeper.

(d) With the A and B values closer to 0, the slope of the end graph is smaller.

(e) As A and B converge to 0, the slope of the graph decreases. Thus the graph does approach the homogenous solution as the forcing function converges to 0.

(f) When A and |B| get larger, the end slope of the graph gets steeper.

(g) Using the exponential initial condition, there is no significant change in the graph evolution.

(h) The step function initial condition also makes no significant change in the graph evolution.

(iv) This is still the Neumann boundary conditions.

(a) The graph evolves to a plane, which immediately begins rising. This is the 2D version of a rising line, which is the 1D non-homogeneous case.

(b) With the higher k^2 value, the initial graph evolves faster.

(c) Similarly, the lower k^2 value slows the evolution down. With $k^2 = 0.25$, the graph has not yet reached the plane at t=0.5. It finally does evolve to the plane at about t=1.5.

(d) This graph is very similar to the graph in part (a), except that the end plane is slightly higher with the lower *a* value.

(e) As the *a* value gets lower, the plane at t=0.5 approaches 1.5. This definitely does not approach the homogenous case (which was a static plane at 1).

(f) On the other hand, larger values of *a* make the graph start to converge to the static plane at 1. This makes sense based on the forcing function, e^{-at}. As *a* increases, the forcing function approaches 0.

(g) Similarly to the cosine initial condition, the graph quickly evolves into a plane that shifts uniformly upwards. The homogenous case, however, converges to a plane at approximately 0 and remains there. All other conclusions about varying k^2 and *a* remain the same.

(h) With the step function initial condition, the conclusions about k^2 and *a* are the same as with the cosine and the exponential initial condition cases.

(v)

(a) The graph quickly evolves from a central peak and four exterior troughs to a rising plane. A t continues, it can be seen that this plane oscillates up and down as a whole. In the 1D case, it was a line that oscillated up and down. Thus, this is just the 2D version of the 1D case, with no major changes.

(b) As should be expected, the higher k^2 value does speed the evolution of the graph slightly. For a more noticeable effect, try $k^2=4$.

(c) However, the effect of the lower k^2 value is more noticeable, as it distinctly slows down the evolution of the function.

(d) Graphing this function until around t=5 would be helpful. Lowering ω increase the period of the oscillation and the amplitude of the oscillation.

(e) Similarly, as ω gets smaller, the graph has a larger period and oscillates with higher amplitude. Thus, this certainly does not converge to the homogenous solution of a static plane at 1.

(f) On the other hand, as ω gets larger, the graph oscillates faster with a lower amplitude about 1. This, as ω approaches infinity, would converge to a static plane at 1.

(g) The same conclusions made for the cosine initial condition will hold for the exponential initial condition.

(h) The same conclusions will also hold for the step function initial condition.

MATHLAB Exercise 11.1.6

(i)

(a) With the homogenous case, the solution evolves into a plane at 0. For the non-homogenous case, the graph arcs slowly upward in the center, while the edges remain at 0. The result is a mound-shaped graph. The graph does reach an endpoint, with the graph remaining fixed after a certain point in time.

(b) Lowering A results in a lower peak to the graph at larger t values.

(c) Continuing to lower A causes the peak to get smaller and smaller. As A approaches 0, the peak disappears, and the graph approaches the homogenous solution.

(d) The same results are found with the exponential initial condition.

(e) The same results are found with the step function initial condition.

(ii)

(a) The homogenous case again evolves into a plane at 0. With this non-homogenous case, the graph evolves into a mound that is stable with time, while the edges remain fixed at 0.

(b) The overall evolution of the graph is the same as in part (a), however, the end peak is lower.

(c) As A approaches 0, the peak of the graph gets smaller. Thus, the graph does approach the homogenous solution.

(d) The same results are found with the exponential initial condition.

(e) The same results are found with the step function initial condition.

(iii)

(a) Here, the graph evolves into a sloped plane that then shifts downward with time.

(b) With the smaller A and |B| values, the graph is less sloped and does not shift downward as quickly as in part (a).

(c) Similarly, the smaller A and |B| values cause the graph to less sloped and shift downward slower. Thus, the graph does approach the homogenous solution as A and |B| approach 0. With the larger A and |B| value, the graph becomes a steeper plane, and the graph shifts downwards faster.

(d) The same conclusions can be reached with the exponential initial condition.

(e) The same conclusions can be reached with the step function initial condition.

(iv)

(a) Here, the graph evolves to a plane that then rises to hold steady at slightly below 2.

(b) With the lower A value, the end plane holds steady at a higher value.

(c) As A gets smaller, the graph evolves to a higher plane. With a larger A value, the end plane is much lower. Thus, as A approaches infinity, the graph approached the homogenous solution.

(d) The same conclusions can be reached with the exponential initial condition.

(e) The same conclusions can be reached with the step function initial condition.

(v)

(a) The graph evolves into a plane that oscillates up and down. The period is approximately 6.3 seconds with center at 1 and amplitude of 1.

(b) This graph is similar, with the center of 1. However, the amplitude is now about 2, and the period is approximately 12 seconds.

(c) With a lower A value, the amplitude increases inversely proportional to the A value, and the period also increases. With a higher A value, the amplitude decreases as well as the period.

(d) The same conclusions can be reached with the exponential initial condition. The only difference is the center of the oscillations is 0.

(e) The same conclusions can be reached with the step function initial condition. Again, however, the center of the oscillations is 0.

MATHLAB Exercise 11.1.7

(i)

(a) With the lower k^2 value, the graph evolves slightly faster. Therefore, the final peak is slightly higher. The difference between the two norms is initially slight, but the sup norm is more accurate over long periods of time as they slowly diverge from each other.

(b) With higher k^2 values, the graph evolves slower, and the final peak is lower. With lower values, the graph evolves faster and the final peak is higher. In all cases, the sup norm is still more accurate over the longer periods of time.

(c) With k^2=2, the graph reaches its peak of about 0.72 at around t=0.50. When perturbed to 1.8, the graph reaches its peak of 0.79 at around t=0.60. Watching the graph, it is almost impossible to differentiate between the sup and L2 norms, but the L2 norm does have a greater difference from the original. The perturbation to 1.9, the graph hits its peak of 0.77 at around t=0.57. With 1.99, the graph hits its peak of about 0.73 are around t=0.52. With 2.1, it hits a peak of about .68 at around t=0.45. With k^2=2.01, it hits a peak at about 0.70 at around t=0.58. It is logical to make the conclusion that both the peak and time that the graph reaches that peak varies continuously with the k^2 value.

(d) Here, with the graph so small, it is hard to view the time the graph reaches its peak. However, the peaks are at 0.06 when k^2=2, 0.07 when k^2=1.9, 0.06 when k^2=1.99, 0.06 when k^2=2.01, and 0.05 when k^2=2.1. So here this supports the conclusion of the peak height, although there is not really enough evidence to support the change in peak time.

(e) The forcing function influences the height overall height of the graph at the end solution. The higher the A value, the higher/larger the graph peak, and vice versa.

(f) The same conclusions can be reached utilizing the exponential initial condition.

(g) The same conclusions can be reached using the step function initial condition.

(ii)

(a) With the lower k^2 value, the graph evolves slightly faster. Therefore, the final peak is slightly higher. The difference between the two norms is initially slight, but the sup norm is more accurate over long periods of time as they slowly diverge from each other.

(b) With higher k^2 values, the graph evolves slower, and the final peak is lower. With lower values, the graph evolves faster and the final peak is higher. In all cases, the sup norm is still more accurate over the longer periods of time.

(c) With k^2=2, the graph reaches its peak of about 0.42 at around t=0.50. When perturbed to 1.8, the graph reaches its peak of 0.42 at around t=0.5, but is still rising at the end. Watching the graph, it is almost impossible to differentiate between the sup and L2 norms, but the L2 norm does have a greater difference from the original. The graphs are all still rising at t=0.5, but the peaks are different. Thus, with the perturbation at 1.9, the graph hits its peak of 0.45. With 1.99, the graph hits its peak of about 0.42.

With 2.1, it hits a peak of about 0.4, and with k^2=2.01, it hits a peak at about 0.43. The k^2 value here is not acting continuously with peak height at t=0.5.

(d) Here, the graphs are so small that the peaks are difficult to differentiate their peak heights (as in part (i), the peak heights differed at most by 0.01). So there is not really enough evidence to say whether the graph varies continuously with respect to k^2.

(e) The forcing function, once again, simply altered the height of the peaks of the graph. Higher value of the forcing function gave a higher peak.

(f) The same conclusions still hold using the exponential initial condition.

(g) The same conclusions still hold using the step function initial condition.

(iii)

(a) With the original graph, the end solution is practically a plane, sloped from 0.6 in one corner to 1.2 in another. With a perturbation value of 0.1, the sup norm is a sloped plane with low at about 0.5 and high at 1.1. The L2 norm yields the same low and high.

(b) With the changing perturbation values, the low and highs of the sloped plane vary slightly. For instance, with the perturbation value of 0.05, the low value is about 0.55 and the high is about 1.15. Thus, the perturbation value appears to shift the graph downwards an amount equal to its value. Thus, it is a continuous variation.

(c) With the original graph, it evolves into a sloped plane with low at around -1.15 and high at about 1.9. With the perturbation value at 0.1, the low is around -1.25 and high is at 1.8. Similarly, the perturbation value of 0.05 yields a high at 1.85 and a low at -1.2. Thus, the same conclusion as above can be reached: it is continuous and it causes a downward shift.

(d) The same results as in (b) and (c) can also be seen here, with the perturbation values shifting the graph downwards in a continuous manner.

(e) The forcing function here altered the slope of the graph. With values of 1 and -1, the plane spanned a height of 0.6, regardless of the perturbation. However, with the values of 5 and -5, the graph spanned a height of around 3.05, and with 0.5 and -0.5 a height of around 0.3.

(f) With the exponential graph, altering the perturbation value does not appear to alter the graph at all. Although this is a change from part of our conclusion above, it does not alter the conclusion that the graph varies continuously (no variation is still a continuous variation). This conclusion holds with respect to all the differing forcing function values, and the conclusion of the greater slope/height of the plane also holds with the exponential initial condition as well.

(g) Utilizing the step function does alter the graph slightly. Unperturbed, the graph goes from -0.18 to about 0.42. Using a perturbation of 0.1 drops the graph to about -0.2 to 0.4. Similarly, perturbing the graph with 0.05 yields the end plane with a high at 0.41 and the low at -0.19. Thus, like the cosine initial condition, the perturbation drops the graph proportional to the perturbation value. Therefore it too causes a continuous variation.

(iv)

(a) With the original graph, it evolves quickly into a plane that then shifts upward. It ends at about 1.4. With the perturbation and the k2 change, the graph now ends at 1.3. However, there is a great difference between the L2 and the sup norm. The L2 norm steadies out with a 0.2 difference, whereas the sup norm goes to 0.

(b) Keeping the perturbation the same, altering the k2 value does not vary the graphs that much. It does slightly shift the graphs, but to such a small extent that it is hard to see with the existing values. Again, however, the perturbation value shifts the graph downwards. Thus, both parameters have a continuous effect on the graph. The only effect that can be seen with the k2 value is as the k2 value gets farther from the original, the sup difference takes a bigger jump in the middle of its convergence. This disappears as k2 approaches 2.

(c) The original graph rises to a plane at about 1.2. As expected, with a new k2 value of 1.8 and a perturbation of 0.1, the graph now only rises to about 1.1. The L2 norm converges to a 0.2 difference, and the sup norm converges to 0 difference. Again, the k2 value does not have a great influence on the graph, but the perturbation value does lower the end plane. The k2 value does still cause the bump in the sup difference.

(d) Here the original graph ends at around 1.45. As seen above, the k2 value does not really alter the end position of the graph, but the perturbation value does lower the end solution plane. The k2 value does still cause the bump in the sup difference.

(e) The only effect the forcing function seems to have is causing a bump in the convergence of the sup and L2 norms. The bump is hard to see with the L2 norm (and is most visible with k2=1.8), but as the k2 value gets farther away from 2, the bump is larger.

(f) Again, here the perturbation value drops the graph, while the k2 value does not appear to change the graph. However, the two norms both converge to a value close to 0 with this initial condition, with the L2 value converging to a higher value. The k2 value does alter the point of crossover between the two norms, seeming to shift it to the left as k2 approaches 2.

(g) Again, here the perturbation value drops the graph, while the k2 value does not appear to change the graph. However, the two norms both converge, with the L2 value converging to a higher value. The k2 value does alter the point of crossover between the two norms, seeming to shift it to the left as k2 approaches 2. Also, the k2 value does effect the amount of bump in the graph.

(v)

(a) Here, the end graph is again lowered a small amount, and the change in omega cannot be differentiated from keeping it the same. The difference in the norms appear to approach the 0.2 and 0 for the L2 and sup norms, respectively, but the lines seem to start increasing near t=0.5. Graphing it out to t=2, it can then be seen that those norms both diverge to infinity, with the sup norm still always less than the L2 norm.

(b) When perturbing all these values, it is known from the prior problems that the k2 value barely alters the graph and the perturbation value lowers the graph. With regards to the norm, the perturbation value alters where the norms converge to and where the two norms cross, and the k2 value causes a bump in the graph. Also, the perturbation value seems to also control the size of the k2 bump. Now, adding in the cosine forcing function alteration, the forcing function does not appear to alter the graph itself. However, it does clearly alter the norm graphs. Altering this function changes the end solution of the norm graphs. Leaving it the same allows the graph to converge to a non-zero value, as described in previous problems. Perturbing the omega value higher causes the two norms to both diverge to infinity. Conversely, perturbing it lower causes the graphs to initially perform the same, but then to converge to 0 rather quickly (try graphing it for times up to t= 1.5 just to see). Thus, it is clear that the perturbation value and the k2 value are continuous from previous problems. With the omega value, it does seem to be continuous with respect to the graph, though it is clearly discontinuous (at least at omega=1) for the norms.

(c) Here again, the actual graph does not appear to be changed by the omega value. Graphing it out to longer times, however, does show the oscillations speed or slow by changing omega, and the height and speed altered by the k2 value, but in t=0.5 this difference is not noticeable. Contrary to (a) and (b), when altering the omega value, both norms will always diverge to infinity, though still with the L2 norm greater than the sup norm. Most other conclusions remain the same, however. The graph still remains continuous with respect to all three variables: perturbation, k2, and omega.

(d) The graph itself is once again practically unaltered by the k2 and omega perturbations, while the perturbation value itself lowers the graph. All the other conclusions reached in part (b), however, apply to the step function initial condition.

(vi) All of the variables explored in these problems continuously effect the graph. The k2 value generally effects the speed and/or height of the graph, whereas the perturbation value generally lowered the graph. The forcing function, obviously, had different effects depending on which forcing function was used, but it all did cause a continuous variation. With regards to the norms, the sup norm was by far the more accurate, consistently having a lower difference than the L2 norm. However, it was visible in some graphs that for short times and lower perturbation values, the L2 norm was more accurate. Thus, the choice for which norm would largely depend on the perturbation value and the length of time. A short time, small perturbation value would imply an L2 norm being accurate, whereas a long time or a large perturbation value would imply the sup norm.

MATHLAB Exercise 11.2.1

(i) This is a Dirichlet boundary condition.

(a) The graph starts out as a sine graph, and the amplitude steadily decreases. By t=0.5, the graph is still a sine graph, but the amplitude is much lower. It is also slightly upward shifted, as the point where the graph crosses the x-axis has shifted slightly to the left, and the peak is higher than the trough is low. This is different from the homogenous case, where the graph remains an odd function about the y-axis.

(b) As expected, lowering Beta (permeability) slows the evolution of the graph significantly. It still has the slightly lopsided evolution with the trough rising faster than the peak drops, but it does so at a much slower pace than in part (a).

(c) Raising Beta speeds up the evolution of the graph, still with the lopsided evolution as in part (a).

(d) Lowering the A value reduces the evolution of the graph towards the positive end. The peak on the right side is not as high, and the left trough is lower than the A=1 graph. It is, however, still different than the homogenous case. Thus, A=0.5 is partway between the A=1 graph and the homogenous case.

(e) As A gets smaller, the difference between the homogenous case and this non-homogenous case decreases. With A=0.001, the two are practically indistinguishable. Thus, the graph does vary continuously based on the forcing function parameter, and therefore it does converge to the homogenous case.

(f) As A gets larger, the peak lowers much slower, and the trough raises much quicker. With A=5, the peak barely drops, whereas the trough gets almost to 0 by t=0.5. Graphing for longer times, the graph eventually evolved to a single peak centered at 0. The higher the A value, the faster the graph evolves to this end point.

(g) Here, with the initial condition a single peak, the graph still evolves to a single, sinusoidal-like peak. With the homogenous case, the sides of the graph rise only slowly (as it seeps in from the central peak), while the central peak drops quickly. The sides of the non-homogenous case rise much quicker to meet the central peak, which drops slowly. Reducing beta actually causes the central peak to rise along with the sides, although the sides rise faster than the peak. Increasing beta causes the central peak to drop much quicker, and the graph actually reaches the sinusoidal end solution by t=0.5 when using Beta=5. Similar to the sine initial condition, the lower A values further slow the rise of the sides and increase the fall of the middle. Higher A values, on the other hand, speed the rise of the sides and slow or even stop the decrease of the central peak. It is still plain, however, that as A approaches 0, the graph approaches the homogenous case. Thus, the conclusions made with the sine initial condition all hold, with just minor modifications.

(h) The step function graph behaves almost exactly like the exponential graphs. All the conclusions reached utilizing that graph hold with the step function graph. The only excepting is with the beta=0.5 graph, the peak still drops, but the end result would be the same: a higher peak than the beta=2 case.

ii)

(a) - (f) The differences between the step function forcing function and the uniform forcing function (i) are practically nill. The same conclusions as seen above can be reached with ease with the fifth (step function) forcing function. The only slight difference is the sides of the graph do appear to lag behind the evolution of the rest of the graph, and thus retard the evolution of parts of the center due to that lag. However, all conclusions still hold from part (i).

(g) Using the exponential function still yields all the same conclusions.

(h) Using the step function still yields all the same conclusions.

iii)

(a) The homogenous case keeps the cosine curves, and the amplitude steadily decreases. With the forcing function, the same occurs, but the peaks and troughs are lower the farther to the left you go. Thus, the highest peak and the highest trough are on the far right, and the lowest is on the far left. If you just graphed the peaks, those peaks would form a line with positive slope, and the same for the troughs. As time progresses, the slope of these two parallel lines increases slowly.

(b) Decreasing alpha speeds the graph up by a large margin. By t=0.5, the graph is practically a single sloped line.

(c) Increasing alpha slows down the evolution of the graph, although the general trend does continue.

(d) As mentioned in part (a), there are two parallel lines that would run through the peaks and the troughs of the graph, respectively. Lowering the absolute values of the parameters of the forcing function makes these lines shallower than normal.

(e) As the values get smaller, the slope becomes harder to notice. Eventually, the graph does approach the homogenous solution.

(f) Conversely, increasing the values makes the slope steeper.

(g) With the exponential initial condition, it is easier to see the graph evolving to the sloped line. As above, alpha controls the speed and the forcing function parameters control the slope of the graph.

(h) All of the conclusions reached above hold for the step function initial condition.