McKibben Webster – Chapter 2 Partial Solutions and Hints

Ex. 2.1.1:

i) P(x) doesn't hold for all x
ii) There exists at least one x such that P(x) doesn't hold

Ex. 2.2.1:

i)
a. If $A \subseteq B$ then, all $x \in A$ are $x \in B$. By contra positive, if $x \notin B$ then $x \notin A$. So, $B^c \subseteq A^c$ as desired.
b. If $B^c \subseteq A^c$ then, if $x \notin B$ then $x \notin A$ and all $x \in B^c$ then $x \in A^c$. By contra positive, if $x \notin A^c$ then $x \notin B^c$ or all $x \in A$ are $x \in B$ and we know that $A \subseteq B$ as desired.

ii)
a. If $x \in A$, then either $x \in B$ or $x \notin B$:
   i. If $x \in B$, then $x \in A$ and $x \in B$. So, $x \in (A \cap B)$ and $x \in ((A \cap B) \cup (A \setminus B))$ as desired.
   ii. If $x \notin B$ then $x \in A$ but, $x \notin B$. So, $x \in (A \setminus B)$ and $x \in ((A \cap B) \cup (A \setminus B))$
b. If $x \in ((A \cap B) \cup (A \setminus B))$ then either $x \in (A \cap B)$ or $x \in (A \setminus B)$
   i. If $x \in (A \cap B)$ then $x \in A$ and $x \in B$. So, $x \in A$ as desired.
   ii. If $x \in (A \setminus B)$ then $x \in A$ and $x \notin B$. So, $x \in A$ as desired.

iii)
a. If $x \in (A \cap (B \cup C))$ then $x \in A$ and either $x \in B$ or $x \in C$. So, either $x \in A$ and $x \in B$ or $x \in A$ and $x \in C$. So, $x \in ((A \cap B) \cup (A \cap C))$
b. If $x \in ((A \cap B) \cup (A \cap C))$ then either $x \in A$ and $x \in B$ or $x \in A$ and $x \in C$. So, $x \in A$ and either $x \in B$ or $x \in C$. So, $x \in (A \cap (B \cup C))$.

iv)
a. If $x \in (A \cap B)^c$, then $x \notin (A \cap B)$ and either $x \in A$ and $x \notin B$ or $x \notin A$ and $x \in B$ then $x \in (A^c \cup B^c)$ $\quad B \not\subset A$
b. If $x \in (A^c \cup B^c)$ then either $x \notin A$ or $x \notin B$, either way, $x \notin (A \cap B)$. Thus, $x \in (A \cap B)^c$

v)
a. If $x \in (A \cup B)^c$ then $x \notin A$ and $x \notin B$. So, $x \in A^c$ and $x \in B^c$, then we know that $x \in (A^c \cap B^c)$
b. If $x \in (A^c \cap B^c)$ then $x \in A^c$ and $x \in B^c$. So, $x \notin A$ and $x \notin B$, then we know that $x \in (A \cup B)^c$

Ex. 2.2.2:

To show that $A \neq B$, we either show that $A \not\subset B$ or that $B \not\subset A$, by showing if $x \in A$ then $x \notin B$ or vise versa.

Ex. 2.3.1:

If dom(f)=dom(g) and rng(f)=rng(g), then the functions f and g are equal.

Ex. 2.3.3:

Let $f(x) = x^2$ and $g(x) = x+1$ then;

$f(g(x)) = (x+1)^2 = x^2 + 2x + 1$;

$g(f(x)) = x^2 + 1$

Thus, for $x \neq 0$ we know $f(x) \neq g(x)$

Ex. 2.3.3:

i) Let dom(f) =x and dom(g)=A then since f is onto, then there exists $a \notin A$ such that f(x)=a. Then g(f(x))=g(a) and since g is onto, then there exists $b \notin B$ such that for some $a \notin A$, f(a)=b. Then g(f(x))=f(a)=b and $g \circ f(x)$ is onto as desired.

ii) If f(f(a))=f(g(b)), then since f is one-to-one, g(a)=g(b) and because g is one-to-one a=b. So f(g(x)) is one-to-one as desired.

Ex. 2.3.4:

i) $\min(S) = -\infty$ for all $x \leq y$

ii) $(f-g)(x) \leq (f-g)(y)$ for all $x \leq y$

iii) For $\frac{f}{g}(x)$ let $f(x) = x-1$ and $g(x) = \frac{1}{x^2-1}$ then $\frac{f}{g}(x) = \frac{x\ -1}{x^2-1} = \frac{(x-1)}{(x-1)(x+1)} = \frac{1}{x+1}$ and the inequality doesn't hold.

iv) For $g \circ f(x)$ let f(x) and g(x) be the same as in (iii) then $g \circ f(x) = \frac{1}{(x-1)^2-1} = \frac{1}{x^2-2x+1-1} = \frac{1}{x^2-2x}$ and the inequality doesn't hold.

Ex. 2.4.1:

i) X=3

ii) X=-8,-4

Ex. 2.4.2:

i) $\max = \varnothing$, sup=14, $\min = \varnothing$, inf=-1
ii) $\max = \varnothing$, $\sup = \infty$, min=2, inf=2
iii) max=2, sup=2, $\min = \varnothing$, inf=1
iv) $\max = \varnothing$, $\sup = \infty$, $\min = \varnothing$, inf=0
v) $\max = \varnothing$, $\sup = \varnothing$, $\min = \varnothing$, $\inf = \varnothing$
vi) $\max = \varnothing$, sup=1, $\min = \varnothing$, inf=-1
vii) $\max = \sqrt{2}$, $\sup = \sqrt{2}$, $\min = \sqrt{2}$, $\inf = -\sqrt{2}$
viii) $\max = \varnothing$, sup=1, min=0, inf=0

Ex. 2.4.3:

i) By showing $\max(S) = \infty$
ii) By showing $\min(S) = -\infty$

Ex. 2.5.1:

If x_n is nondecreasing , then $x_n \leq x_{n+1}$ for all $n \in \mathbb{N}$ there for $\inf(x_n)=x_1$

If x_n is nonincreasing, then $x_n \geq x_{n+1}$ for all $n \in \mathbb{N}$ there for $\sup(x_n)=x_1$

Ex. 2.5.2:

i) Decreasing, bounded above by 1/3
ii) Bounded above by 1 and below by -1
iii) Bounded above and below by 1 and -1
iv) Is bounded below by 16/9 and is increasing
v) Bounded below by -4 and above by 1.

MatLab-Ex. 2.5.1:

i)

a. Works
b. Yes, since $\left|\frac{\sin(1)}{1}\right| < \left|\frac{\sin(50)}{50}\right|$
c. $\left|\frac{\sin(500)}{500}\right| < \left|\frac{\sin(600)}{600}\right|$
d. N=9
e. ε=0.01, N=989; ε=0.0001, N=9996

Ex. 2.5.3:

Because if $\lim_{x \to \infty} x_n = \infty$ then $|x_n - \infty| > \varepsilon$ for all $\varepsilon > 0$

E^n, n^n, and 2^n

True

For all M<0, there exists $N \in \mathbb{N}$ such that when $n \geq N$, it is the case $x_n \leq M$.

MatLab-Ex. 2.5.2:

i)

a. Works
b. Yes
c. N=102
d. M=200, N=202; M=500, N=502
e. N=M+2

ii)

a. $\lim_{n \to \infty} -n(\cos(n)+2)^2 = -\infty$
b. N=92
c. M=200, N=99; M=300, N=287; M=500, N=488; M=400, N=400
d. No, the second equation is oscillating

Ex. 2.5.4:

i) There's only 1 limit for a convergent sequence

ii) If $x_n \to x$, there exists $y, z \in \mathbb{R}$ such that $y > x_n$ and $z < x_n$ for all $n \in \mathbb{N}$

iii) If x_n is bounded above and below by 2 convergent sequences that converge to the same point then $x_n \to p$

a. $x_n + y_n \to L + M$
b. $x_n y_n \to LM$

$$\frac{x_n}{y_n} \to \frac{L}{M}$$

Ex. 2.5.5:

i) Theorem 2.5.1 (v)(b)

ii) Theorem 2.5.1

Ex. 2.5.6:

Y_n is bounded, so there exist a and b such that $a<y_n$ and $y_n<b$ for all $n \in \mathbb{N}$, then y_n converges by the squeeze theorem.

Ex. 2.5.7:

i) If limit of x_n is L then the absolute value of the limit of x_n is the absolute value of L
ii) The limit doesn't exist

Ex. 2.5.8:

i) Since s_n is a sum of positive real numbers, s_n is nondecreasing. If s_n is bounded above then by theorem 2.5. (vi) converges. If it has an upper bound if s_n converges to s, then s_n is bounded by theorem 2.5. (ii)

ii) There exists $N \in \mathbb{N}$ such that $a^N<N!$ for all $n \geq N$ thus $\lim_{n\to\infty} \frac{a^N}{N!} = 0$

MatLab –Ex. 2.5.3:

i)

a. $\varepsilon=0.01$, N=194; $\varepsilon=0.001$, N=1988
b. Done