MAT 161-03—Exam #3—11/16/10

Name: _____

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (20 points) Consider the function $f(x) = x^5 - 5x^4$.

   (a) Determine the intervals on which f is increasing/decreasing.

   (b) Determine the intervals on which f is concave up/concave down.

   (c) Sketch a graph of the function, clearly labeling the coordinates of all intercepts, local extrema, and inflection points.

2. (12 points) Evaluate each of the following limits. Show all work using correct notation!

   (a) $\lim_{x\to\infty} \dfrac{\ln x}{x}$

   (b) $\lim_{x\to 0} \dfrac{1-\cos 2x}{1-\cos 3x}$

3. (8 points) Find the equations of all horizontal and vertical asymptotes of the function $f(x) = \dfrac{45-5x^2}{3x^2-12}$.

   HORIZONTAL:

   VERTICAL:

4. (20 points) A plane is traveling at a constant altitude of 3 miles and a constant speed of 240 mph. At noon, the plane passes directly over a radar station.

   (a) How fast is the distance between the plane and the radar station changing at 12:01 pm?

   (b) How fast is the angle between the vertical and the station's line of sight to the plane changing at 12:01 pm? Give your answer in radians per minute.

5. (8 points) Find the absolute maximum and minimum values of $f(x) = x - 2\sin x$ on the interval $[0, \pi]$.

6. (12 points) Evaluate each of the following indefinite integrals.

   (a) $\displaystyle\int \left(x^5 + e^{5x} + 3\right) dx$

   (b) $\displaystyle\int \left(\frac{3}{x} - \frac{6}{x^3} + 4\cos 7x\right) dx$

7. (20 points) You are asked to design a box of volume 6 cubic feet with square base and no top. The material for the bottom costs \$3 per square foot, and the material for the sides costs \$2 per square foot. Find the dimensions of the box that minimize the total cost, and justify that your solution gives a minimum.