MAT 261—Exam #1—9/18/14

Name: ____

Calculators are not permitted. Show all work using correct mathematical notation.

1. (10 points) Find parametric equations for the line containing the points $(1, 0, 2)$ and $(3, 1, 5)$.

2. (15 points) A particle moves in space with trajectory

$$x(t) = \ln(t^2 + 1), \quad y(t) = \sin(\pi t), \quad z(t) = \frac{3}{t^2}.$$

Find the unit tangent vector (that is, the unit vector tangent to the particle's path) at $t = 1$.

3. (10 points) Convert the spherical coordinates $(\rho, \theta, \phi) = (4, 3\pi/4, \pi/6)$ into rectangular coordinates (x, y, z).

4. (15 points) Find the length of the curve $c(t) = (2t^{3/2} + 1, 5t + 3)$ on the interval $0 \leqslant t \leqslant 1$.

5. (10 points) Find an equation for the plane passing through the origin and containing the vectors $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k}$.

6. (15 points) Consider the points $P(0,1)$, $Q(1,3)$, and $R(1,5)$ in $\mathbb{R}^2$.

   (a) Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Find the area of the triangle with vertices P, Q, and R.

7. (15 points) Consider the plane $x + 2y + 3z = 4$.

(a) Find a unit vector perpendicular to the plane.

(b) Find the point at which the line $\mathbf{r}(t) = \langle 2 + 3t, 1 + t, 4 - t \rangle$ intersects the plane.

8. (10 points) Find a formula for the speed of a particle moving along the helix

$$\mathbf{r}(t) = \langle kt, A \cos \omega t, A \sin \omega t \rangle.$$

Your formula should involve the constants k, A, and ω and should be given in simplest possible form.