The e/m experiment: Student exploration into systematic uncertainty

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In this work, we convert a common verification lab in the physics curriculum, measuring the charge-to-mass ratio of the electron (e/m), into an investigative lab on systematic uncertainty. The Bainbridge apparatus, commonly used to measure e/m, can have significant systematic uncertainties, leading to large discrepancies with the accepted value. Students were asked to quantify possible systematic uncertainties in the apparatus and correct them. Building upon each other's work from semester to semester, students characterized multiple sources of systematic uncertainty. Not only did the students learn about uncertainty analysis techniques that reveal systematic uncertainties, but they also dramatically improved the accuracy of the apparatus, reducing the discrepancy from 15% to 0.5%. This paper describes a pedagogical approach to exploring unknown systematic uncertainties in an intermediate laboratory setting and the student-learning benefits of such an approach. In addition, it provides detailed information about untangling and correcting the sources of systematic uncertainty in the Bainbridge apparatus.

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I. INTRODUCTION

In experimental physics, there are three main sources of uncertainties: random, reading, and systematic. In instructional labs, students learn about estimating random uncertainties from repeated measurements and estimating reading uncertainties from a single measurement of a digital or analog meter.[1] Explicit exploration of systematic uncertainty gets less attention. However, systematic uncertainties often represent the limiting factor in making accurate measurements. Many scientists have devoted a considerable portion of their careers to tracking down and reducing systematic uncertainties.

The Bainbridge apparatus, used to measure the charge-to-mass ratio (e/m) of an electron, can be found in many physics departments around the world.[2] However, it is often plagued with systematic uncertainties.[3,4] Careful and diligent students who use the apparatus to measure e/m will observe discrepancies between the measured and accepted values as large as 10%–15%. After propagating uncertainties, they realize random and reading uncertainties do not account for these discrepancies. It becomes evident to these students that something is amiss.

In a laboratory course, such situations are often addressed by the instructor. They investigate the sources of systemic uncertainty and either inform the students of the results, so they may correct their data, or provide the students with a clear set of procedures to characterize the systematic uncertainties as part of the experiment. However, in this work, we explore a third approach: student exploration. Students were asked to investigate, quantify, and interpret unknown systematic uncertainties in our Bainbridge apparatus. This work spanned multiple semesters, with subsequent students building upon the work of previous students. The instructor encouraged student independence, acting more as a research advisor than a laboratory instructor. At the end of each semester, students presented their results and defended them in front of an audience of faculty and colleagues. This approach aligns well with recent findings from physics education research on laboratory practices and the need to shift the focus of experiments from verification to investigation.[5,6]

In addition to learning about analysis techniques that reveal systematic uncertainties, students also dramatically improved the accuracy and precision of the Bainbridge apparatus. Prior to applying corrections, the average measured value of e/m from 42 measurements was $(2.03 \pm 0.03) \times 10^{11}\,\mathrm{C/kg}$, which is not in agreement with the accepted value of $1.7588 \times 10^{11}\,\mathrm{C/kg}$. The discrepancy of the measured value is 15% and is within 10σ (10 standard deviations) of the accepted value. After applying corrections, the averaged value was $(1.751 \pm 0.007) \times 10^{11}\,\mathrm{C/kg}$. This measured value has a discrepancy of 0.5% with the accepted value and is within 1σ. Moreover, the standard deviation of the measurements was reduced from 0.03×10^{11} to $0.007 \times 10^{11}\,\mathrm{C/kg}$, a factor of four improvement in precision.

The paper is structured as follows: In Sec. II, the physics behind the Bainbridge apparatus is reviewed and uncorrected measurements are reported. In Sec. III, the pedagogical reasoning behind exploring systematic errors is discussed. In addition, some practical details about incorporating this approach into a laboratory course are shared. In Sec. IV, student-designed experiments to explore systematic uncertainty are presented with results. In Sec. V, the physical basis of each systematic uncertainty is scrutinized and discussed. Finally, in Sec. VI, the main conclusions of our work are reviewed.

II. BAINBRIDGE APPARATUS: MEASURING e/m

In 1938, K. T. Bainbridge designed an apparatus for an instructional setting to measure the specific charge of the electron (i.e., the charge-to-mass ratio).[2] Eighty-five years later, this apparatus is ubiquitous in physics departments around the world. For this work, we utilized an apparatus sold by NADA Scientific.[7] The apparatus consists of a

Helmholtz coil surrounding a glass bulb that contains a fixed phosphorescent ruler, an electron gun, and low-pressure helium vapor. The helium vapor glows after excitation by moving electrons. A photo of the apparatus in operation is shown in Fig. 1(a).

The relevant physics of the apparatus is shown in Fig. 1(b). Current (I) in the Helmholtz coil generates a magnetic field (magnitude B) around and within the glass bulb. The electron gun inside the glass bulb consists of a cathode that releases thermions (electrons) after being heated by a filament. These electrons are subsequently accelerated through a potential difference (V) with the anode. Some exit through a small hole in the anode and undergo circular motion due to the centripetal Lorentz force created by the magnetic field. These electrons collide with and excite the low-pressure helium vapor inside the bulb, leading to a bluish-fluorescent path. A phosphorescent ruler fixed inside the bulb is used to measure the diameter ($2r$) of the path.

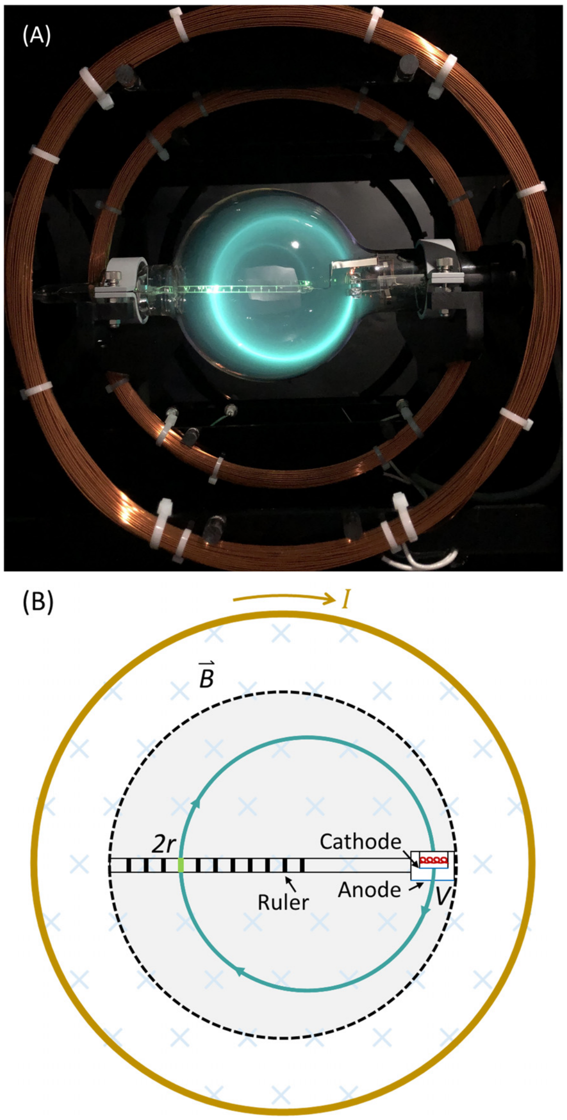

Fig. 1. (a) Picture of NADA Scientific Bainbridge apparatus in operation. (b) Diagram of the apparatus. I is the current in the Helmholtz coil that generates a uniform magnetic field (magnitude, B) at the center of the coil. The internal ruler is used to measure the diameter ($2r$) of the electron path. The accelerating potential (V) accelerates electrons from the heated cathode to the anode plate.

Using the Law of Conservation of Energy, the gain in kinetic energy of the electrons accelerated from the cathode to the anode is eV, where e is the fundamental charge of the electron. The velocity of the electrons passing through the anode plate is thus

$$v = \sqrt{\frac{2eV}{m}}, \tag{1}$$

where m is the rest mass of an electron. Newton's second law expressed for electrons undergoing circular motion by the centripetal Lorentz force is

$$m\frac{v^2}{r} = evB. \tag{2}$$

Substituting Eq. (1) into Eq. (2) and rearranging, we arrive at the well-known electron charge-to-mass ratio equation:

$$\frac{e}{m} = \frac{2V}{B^2 r^2}. \tag{3}$$

In this expression, the accelerating potential (V) is set and measured externally. The radius (r) is determined from measurements of the diameter taken with the internal ruler. The magnetic field strength (B) is calculated from the current (I) in the Helmholtz coil using the Biot–Savart Law. For an ideal Helmholtz coil,

$$B = \left(\frac{8}{5\sqrt{5}}\frac{\mu_0 N}{R}\right)I, \tag{4}$$

where μ_0 is the permeability of free space, N is the number of turns of wire in each coil, and R is the radius of each coil as well as the distance between them. As specified in the manual for the apparatus, $N = 130$ turns and $R = 15.0\,\text{cm}$, therefore $B = (0.779\,\text{mT/A})I$.

The original paper by Bainbridge states an accuracy for individual measurements of 2%. However, the experience of students using the NADA apparatus suggests a far less accurate result. Table I shows collected data for a single measurement with the resultant e/m value. The discrepancy between this e/m value and the stated NIST value (1.7588×10^{11} C/kg) is 15% and exceeds 4σ. Repeating the measurement for a range of magnetic fields, accelerating potentials, and radii resulted in an average value of $(2.03 \pm 0.03) \times 10^{11}$ C/kg, within 10σ of the accepted value.

III. PEDAGOGICAL APPROACH

A likely culprit for these large and repeatable discrepancies is systematic uncertainty, but how should this be handled in a teaching laboratory? We decided to adopt the strategy proposed by Emily Smith and Natasha Holmes in their paper *Best Practices for Instructional Labs*.[6] They identified two critical features for investigation labs: "(1) let students engage

Table I. Preliminary data and resultant value of e/m.

B (mT)	r (cm)	V (V)	e/m (C/kg)
1.309 ± 0.002	4.5 ± 0.1	350.0 ± 0.6	$(2.02 \pm 0.06) \times 10^{11}$

in the decision-making of experimental physics and (2) remove all verification goals." Rather than viewing the systematic uncertainties as a hindrance to a verification lab, we recast them as opportunities for students to engage in authentic experimental exploration. Since the lab instructor had not yet characterized the extent of the systematic uncertainties, the students were not "verifying" a known result but instead generating new knowledge. In addition, no procedure was provided to the students, so they partook in experimental "decision-making" by devising and then revising their experimental approaches to characterizing the uncertainties. Thus, the two features outlined by Smith and Holmes were satisfied.

The overall design of the course is as follows. All students first complete a series of traditional verification labs that focus on teaching data collection techniques as well as approaches to data analysis. Every student is then assigned a different research question that explores an apparatus in the course at a deeper level. Questions, sometimes based upon previous students' work, are selected such that the student is generating new knowledge for the department. For example, with respect to the Franck–Hertz apparatus, a possible research question is "What is the optimal retarding potential for conducting the experiment?" For the Bainbridge apparatus, questions focused on identifying and quantifying the systematic uncertainties in the apparatus. Students designed and conducted experiments to answer their assigned question, explored the relevant science underpinning their results, and then defended these results in a ten-minute, conference-style presentation to the department.

The exploration of systematic uncertainties in the Bainbridge apparatus was more challenging than initially expected. Previous published work on the topic assumed systematic uncertainty is present in only a single variable, such as the magnetic field.[8,9] However, if systematic errors are present in multiple variables, untangling them is more challenging and must be done with care. What started as a simple question, expected to be answered by a single student in a single semester, became a multi-student exploration spanning multiple semesters.

As different students progressed each semester, their assigned research questions became more refined and focused on particular elements of the apparatus. Students were provided with relevant literature and previous students' findings and were then asked to build upon those results. They were also encouraged to reach out to previous students for clarification. Mirroring knowledge creation in the sciences, the students' progress had many twists, turns, and dead ends as they explored the systematic uncertainty of the Bainbridge apparatus. Sections IV and V describe the experiments designed by the students as well as their approach to data analysis to reveal systematic uncertainties.

IV. EXPLORATION OF SYSTEMATIC UNCERTAINTIES

Students identified and explored six possible sources of systematic uncertainty: (1) the current-to-magnetic field conversion factor, (2) the presence of external magnetic fields, (3) the nonuniformity of the magnetic field within the Helmholtz coil, (4) the mispositioning of the internal ruler, (5) the difference between the applied accelerating potential and that experienced by the electrons, and (6) the impact of relativistic speeds of the electrons. The results of these explorations are presented below. The reader should note that the order in which the systematic uncertainties (considered "systematic errors" after characterization) were explored is important since determining the error in a particular variable sometimes depended upon other variables first being corrected. To improve clarity, a "$\sim$" is placed above the corrected variable.

A. Systematic uncertainty in the Helmholtz coil magnetic field measurement

1. I-to-B conversion factor

As discussed in Sec. II, the Helmholtz coil in the apparatus should generate a magnetic field proportional to the current with an I-to-B conversion factor of $0.779\,\mathrm{mT/A}$. To test this conversion factor, the magnetic field produced by the NADA Scientific Helmholtz coil was empirically measured using a magnetometer (524 0381, LD Didactic GmbH). However, the magnetometer's measurement error (2% of the measured value + 0.5% of the range limit value) was too large for the accuracy and precision needed. Students improved the accuracy to better than 0.3% by calibrating the meter using a well-characterized (tightly and uniformly wound) coil (555 604, LD Didactic GmbH). The expected magnetic field generated by the well-characterized coil was calculated using an expression that accounts for the thickness of the coil.[10]

Using the calibrated magnetometer, students measured the magnetic field at the center of the NADA Helmholtz coil as the current through the coil was varied from 0 to 2 A. The relationship between the current and the generated magnetic field was analyzed using linear regression. The I-to-B conversion factor, obtained from the slope of the regression line, was $0.800 \pm 0.005\,\mathrm{mT/A}$. This conversion factor has a 2.6% discrepancy with the value stated in the manual (and calculated in Sec. II), $0.779\,\mathrm{mT/A}$. This, in turn, results in a systematic error of 5.2% in the measurement of e/m.

2. External magnetic fields

The contribution of external magnetic fields was ignored in the derivation of e/m in Sec. II. However, it is well documented in the literature that external magnetic fields, like the Earth's magnetic field, should be accounted for when measuring e/m.[2,8,9] Ultimately, the magnetic field experienced by electrons in the glass bulb of the Bainbridge apparatus is a combination of the magnetic field generated by the Helmholtz coil (B_h) and an external magnetic field (B_e) along the direction of B_h. To accurately determine the net field experienced by the electrons, the unknown B_e must be determined.

Students positioned the apparatus in a location in the lab where the external magnetic field was minimal. The field was surveyed using a 3-axis digital magnetic field sensor common in many smart phones.[11] The sensor's readings were accessed and displayed using the Physics Toolbox Sensor Suite application.[12] The external magnetic field varied significantly (roughly between $10\,\mu\mathrm{T}$ and $110\,\mu\mathrm{T}$) throughout the room, providing strong evidence of magnetic field sources in addition to the Earth's field.

Next, two sets of measurements were taken. For both sets, V was varied from 250 to 450 V in steps of 50 V as measured by a Fluke 87 V multimeter (stated accuracy, 0.05%). For the first set, the apparatus was aligned with B_h parallel with B_e. At each accelerating potential, the current in the Helmholtz coil was adjusted to vary the radius of the beam. The current was chosen such that the radius varied from 3.0 to 5.5 cm in 0.25 cm increments. For the second set of data, the apparatus was rotated 180° such that B_h and B_e were antiparallel. In this position, measurements were made using the same V and r values as used in the first set. However, because of the opposing contribution by the external magnetic field, a greater current was needed to produce these same V and r values. For example, consider a beam path of 5.5 cm radius in an accelerating potential of 250 V. In the parallel orientation, the current needed was 1.115 A. In the antiparallel orientation, the current needed was 1.167 A. Therefore, to accurately determine the magnetic field experienced by the electrons, the external magnetic field must be known.

In each orientation, B_h is calculated from the current in the Helmholtz coil. Let $B_{h,\uparrow\uparrow}$ be the value of B_h when parallel with B_e and $B_{h,\uparrow\downarrow}$ be that when antiparallel with B_e. Continuing with the example above, $B_{h,\uparrow\uparrow} = 900$ and $B_{h,\uparrow\downarrow} = 909\,\mu$T. Since B_e differs only by a sign after apparatus rotation, $B_{h,\uparrow\uparrow}$ and $B_{h,\uparrow\downarrow}$ should differ by $2B_e$. This leads to a B_e of about 5 μT for the example values. In general, for a particular value of V and r, the horizontal component of the external magnetic field can be calculated using the following expression:

$$B_e = \frac{B_{h,\uparrow\downarrow} - B_{h,\uparrow\uparrow}}{2}. \tag{5}$$

Performing this analysis for all stated combinations of radii (3.0–5.5 cm in 0.25 cm increments) and accelerating potentials (250–450 V in 50 V increments), the average external magnetic field was determined to be $B_e = 6.6 \pm 1.4\,\mu$T. This represents about 0.5% of the generated Helmholtz coil field, or approximately 1% systematic error in the calculation of e/m.

3. *Nonuniformity of the Helmholtz magnetic field*

A Helmholtz coil consists of two identical circular coils connected in series, mounted coaxially, and separated by a distance equal to the radius of the coils. A current through the coils generates a uniform magnetic field at the center of the mid-plane, the plane parallel with the planes of the coils and halfway between the two coils. However, the electrons moving in the experiment are never at the center of the mid-plane. It is well known that the magnetic field strength decreases with increasing radial distance from the central axis.[13,14] This variation can contribute to the systematic uncertainty in the magnetic field strength.

To determine the significance of this uncertainty, students measured the magnetic field in the mid-plane as a function of radial distance from the central axis using the calibrated Leybold meter. They compared these measurements with an analytical expression for the variation of the magnetic field with radial distance.[15] To within uncertainty, the experimental data matched the analytical expression, as shown in Fig. 2. Based on the analytical expression over the relevant experimental range of radii, the magnetic field strength should change by less than 0.9%. However, correcting for this systematic error would be nontrivial since the circular path of the electrons is not concentric with the Helmholtz coil for all radius values (see Fig. 1). Given the complexity of the correction and the limited impact on the magnetic field based on experiment and theory, students chose not to correct for this systematic error.

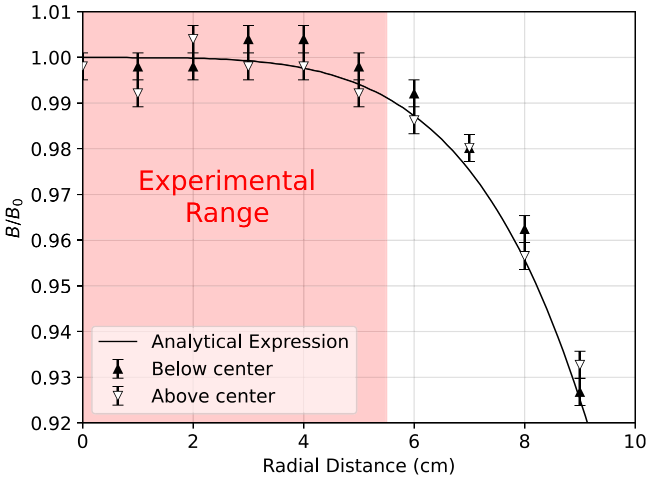

Fig. 2. The decrease in the magnetic field with increasing radial distance from the central axis of the Helmholtz coil. B_0 was the central field value. Error in the measured B values was determined from the precision of the measuring device. The solid line represents the analytical expression. The shaded region is the radial distance range used while measuring e/m. The inner radius of the coils was roughly 15 cm.

B. Systematic uncertainty in radius measurement

With the magnetic field of the apparatus well characterized, students turned their attention to the positioning of the phosphorescent ruler inside the glass bulb. After examining Eq. (3), students realized that at constant V, $r \propto 1/B$ and any offset in this relationship would indicate a systematic error in the radius. It is important to note that the presence of an external magnetic field must be corrected in order for this offset to be attributed entirely to r.

The relationship between r and $1/\tilde{B}$ was plotted for five different accelerating potentials; identical measurement data ranges were used here as in Sec. IV A 2. Figure 3 shows the relationship as well as linear regression lines fit to the data. The y-intercepts (inset of Fig. 3) were averaged and the average was used as the systematic error in the radius,

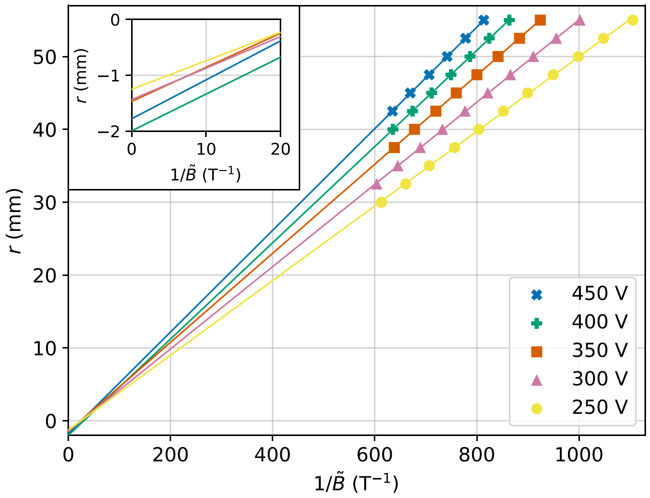

Fig. 3. Relationship between the electron path radius and the inverse of the magnetic field strength at five accelerating potentials. The solid lines are linear regression fits. INSET: The average of these y-intercepts was used as the systematic error in the radius and was determined to be $\delta r = -1.6 \pm 0.2$ mm. $\tilde{B}$ is the corrected magnetic field.

determined to be $\delta r = -1.6 \pm 0.2$ mm. This offset represents a systematic error of 3%–5% in the measured r values and 6%–10% for the measured value of e/m. Note that 1.6 mm is larger than the width of the electron beam, suggesting the uncertainty is not due to a systematic reading error, but rather a ruler misalignment.

Although not explored in this work, students could try alternative approaches to measure the radius. For example, a cathetometer could be used to measure the diameter of the electron's path[16,17] or images of the orbit could be analyzed using imaging processing techniques.[3,4] Both of these approaches have the added benefit of not relying on an accurate measurement of the magnetic field.

C. Systematic uncertainty in accelerating potential measurement

Another source of systematic uncertainty explored by the students was the accelerating potential (V) of the electrons in the electron gun. Referring back to Eq. (3), $V \propto B^2 r^2$. After accounting for the external field(s) and the systematic error in the radius, the systematic error in V was revealed by analyzing the linear relationship between V and $\tilde{B}^2\tilde{r}^2$.

The relationship between V and $\tilde{B}^2\tilde{r}^2$ was plotted using the measurement data ranges in Sec. IV A 2. The relationship is linear and well-fit by a regression line, as shown in Fig. 4. Similar to the radius analysis, the y-intercept of the regression line in the figure was used as the systematic error in the potential and was determined to be $\delta V = 3.9 \pm 0.8$ V. This offset represents a systematic error of about 1% in the measured value of the accelerating potential as well as the measured value of e/m.

D. Systematic uncertainty due to relativistic electron momentum

During a presentation of these results, a faculty member asked about the possible impact of the relativistic momentum of the electron. In particular, the accepted value of e/m is based on the rest mass of an electron. In the apparatus, the electrons are accelerated by a potential as high as 450 V. Using content from a modern physics course, a student concluded that the impact was minimal. The calculation suggests that the Lorentz factor for an electron accelerated by 450 V is 1.000 88. For this Lorentz factor, the change in the momentum of the electron due to relativistic effects is less than 0.09% and thus insignificant as compared to other sources of uncertainty.

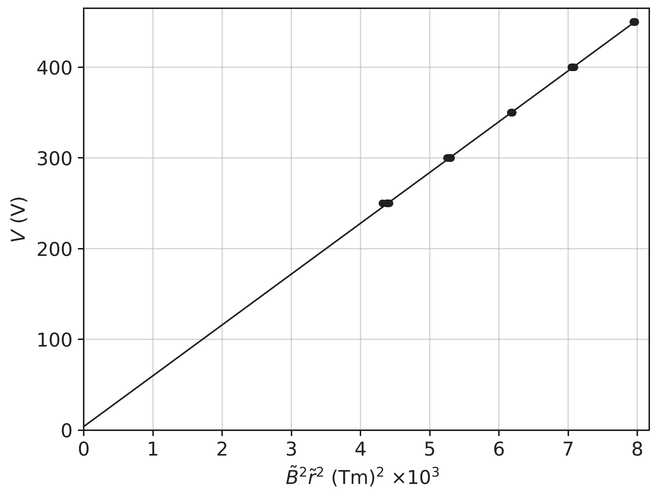

Fig. 4. Relationship between five applied accelerating potentials and the product $\tilde{B}^2\tilde{r}^2$. The solid line is a linear regression fit. The y-intercept of the fit was used as the systematic error in V and was determined to be $\delta V = 3.9 \pm 0.8$ V.

V. DISCUSSION

In this section, we explore the impact of the systematic errors on the measured value of e/m and offer an interpretation on their physical sources.

A. Correcting *e*/*m*

The values from Table I are presented in Table II after accounting for the systematic errors in the magnetic field, radius, and accelerating potential. The measured e/m value and the NIST stated value (1.7588×10^{11} C/kg) now agree and their discrepancy is reduced. The reader should note that all systematic uncertainties were determined *without* utilizing the NIST value of e/m.

As mentioned in Sec. IV, an additional 41 measurements were made by varying the experimental parameters. The accelerating potential was varied from 250 to 450 V in 50 V increments. At each potential, the radius of the beam path was varied from 3.0 to 5.5 cm in 0.25 cm increments by adjusting the current in the coils, which itself varied from about 1 to 2 A. The average of the 42 corrected values of e/m was $(1.752 \pm 0.007) \times 10^{11}$ C/kg, which agrees with the accepted value to within 1σ. The standard deviation of the measurements decreased from 0.03 to 0.007×10^{11} C/kg. In addition, the average discrepancy for an individual measurement was 0.5%, with a maximum of 1.3%. These results are within Bainbridge's stated accuracy of 2% for individual measurements using his original apparatus.[2]

B. Possible sources of systematic errors

After determining a systematic error, students were expected to research possible physical sources for the error. Some were obvious, others less so. This is a summary of the students' analysis, guided by the instructor, for the possible sources.

1. Helmholtz coil magnetic field

The I-to-B conversion factor for the Helmholtz coil was found to be 2.6% larger than that stated in the manual. This is likely due to manufacturing uncertainties in the dimensions of the Helmholtz coil as well as the oversimplified derivation of the relationship between the current and the generated magnetic field.[10] A deviation from the coil radius, 15 cm, by roughly 7 mm would lead to this error. An external magnetic field of 6.6 ± 1.4 μT was measured at the location

Table II. Corrected values $\tilde{B}$, $\tilde{r}$, and $\tilde{V}$, and the resultant value of e/m.

$\tilde{B}$ (mT)	$\tilde{r}$ (cm)	$\tilde{V}$ (V)	e/m (C/kg)
1.352 ± 0.001	4.66 ± 0.10	346.4 ± 0.8	$(1.75 \pm 0.05) \times 10^{11}$

of the apparatus. This is smaller than the horizontal component of the Earth's magnetic field, suggesting shielding effects from the lab building or additional external magnetic fields present in the lab.

2. Internal ruler

The phosphorescent glass ruler fixed inside the bulb was found to be offset by $-1.6 \pm 0.2\,\text{mm}$ from true zero. This represents an offset in the horizontal positioning of the ruler. Since the ruler's incremental markings do not extend across the entire glass rod, as shown in Fig. 1, students assumed that the right edge of the beam's path was intended to correspond to 0 cm, the reference point for diameter measurement. At the time of manufacturing, the ruler was likely inaccurately positioned, leading to a shift in the reference point for the beam path's diameter measurement by $2\delta r$. The ruler could also be offset in the vertical direction, or it could be tilted. These cases were ignored in our analysis but are addressed in Sec. V B 3.

3. Accelerating potential

The accelerating potential was found to be shifted by $3.9 \pm 0.8\,\text{V}$, suggesting the electrons experience a potential that is δV less than the applied potential. Students explored whether this potential drop occurred somewhere along the electron gun circuit by measuring the potential between the cathode and anode of an old glass bulb. They found that no potential drop occurred. A recent paper suggests that the potential experienced by the electrons is reduced by the work function of the cathode.[18] However, we were unable to find a physical basis to support this idea. The work function of the cathode impacts the rate of thermionic emission of the electrons from the cathode's surface (the Richardson–Dushman equation) but has minimal impact on the final velocity of the electrons leaving the anode. Furthermore, tungsten cathodes in electron guns are often coated with barium oxide, which lowers the work function and promotes thermionic emission.[19] For this case, the expected work function is less than 2 eV, which is not in agreement with our measured offset. Searching for other clues, students measured the dependence of the anode current on the acceleration potential (I_a vs. V curve) for the anode plate in the electron gun. This measurement revealed the electron gun was working in the space-charge-limited regime, but it did not provide any insights into the possible offset.[20]

Without a clear physical basis for the 3.9 V offset, students explored other possible sources of it. They found that the uncertainties themselves in the corrections for the magnetic field ($B_e = 6.6 \pm 1.4\,\mu\text{T}$) and the radius ($\delta r = -1.6\ \pm 0.2\,\text{mm}$) were not large enough to account for the systematic uncertainty in the accelerating potential. However, higher-order corrections for the misaligned ruler might be able to account for it. For example, if the ruler is offset vertically (see Fig. 1), the correction to the measured radius is not a constant but instead varies with the radius values.[21] Thus, the apparent systematic uncertainty in the accelerating potential might be a byproduct of an oversimplified correction to the radius. More research is needed to explore this possibility. Directly measuring the diameter of the beam path with a cathetometer or performing image analysis on photos of the electron path might help to unravel this mystery, but we leave this as a potential topic for a future laboratory student to explore.

As a final note, the systematic error in the accelerating potential can be ignored entirely. Without a physical basis for its existence, there is a concern that including it might overcorrect the data. If it is ignored, the measured value of e/m becomes $(1.767 \pm 0.008) \times 10^{11}\,\text{C/kg}$, which is still in agreement with the NIST value to within 1σ with a discrepancy of less than 0.5%.

VI. CONCLUSION

In this paper, we present the findings of students in an intermediate laboratory who investigated sources of systematic uncertainty in a Bainbridge apparatus. Systematic uncertainty is a critical concept in uncertainty analysis and yet is often overlooked in undergraduate laboratory courses. As succinctly stated in Ref. 22: "students often enter University knowing only about systematic uncertainties and leave it thinking only about statistical errors." Shifting the experiment from a verification lab (confirming the value of e/m) to an investigation lab (quantifying unknown sources of systematic uncertainty in the measurement of e/m) bridges the gap between the two and introduces students to the uncertainty analysis techniques focused on revealing systematic error.

In addition, the students reported the experience felt more like authentic research than a traditional laboratory course. Questions about possible sources for uncertainty, experimental procedures and methods, and data analysis occurred organically as students made progress. Students also had the opportunity to engage with literature more than they would have in a traditional intermediate laboratory setting, and they built upon knowledge created by previous students as they would have in real research. The experience gave them a sense of agency as experimentalists. Finally, their efforts greatly improved both the accuracy and the precision of the apparatus. The discrepancy of a single measured value of e/m with the NIST reported value was reduced from 15% to 0.5%. This is a significant improvement overall and within the 2% accuracy stated in Bainbridge's original findings.

The authors hope this work is useful for instructors who wish to evolve verification labs into exploration labs. Although the focus of this paper is on the NADA Bainbridge apparatus, other versions of the Bainbridge apparatus or entirely different experiments known to suffer from systematic uncertainties would work just as well. Whatever experiment is chosen, the instructor must resist the temptation to do the exploration for the students and, instead, let them lead the way.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

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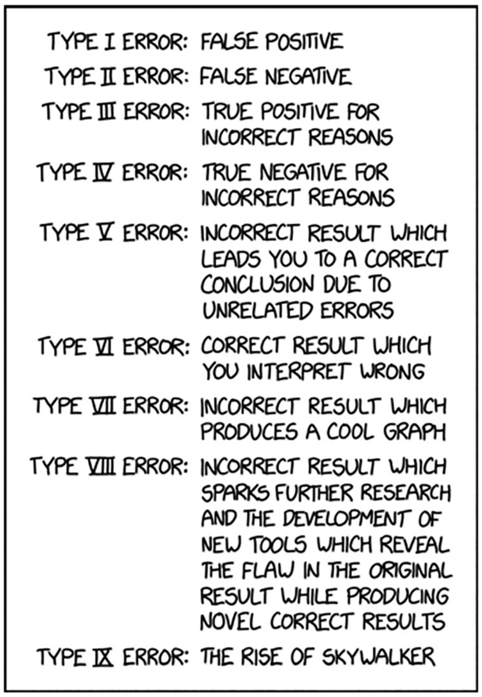

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